

# Ratios and Proportional Relationships

## Domain Overview

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### GRADE 6

Sixth graders are introduced to ratio, a relationship or comparison of two quantities or measures. Students represent ratios in various forms and compare types of ratios. At this level, they use reasoning about multiplication and division to solve ratio and rate problems about quantities. Students learn how and where ratios and rates are used in the real world.

### GRADE 7

Continuing to develop an understanding of operations with rational numbers, seventh graders describe situations in which opposite quantities combine to make zero and determine the absolute value for a given number. Students estimate solutions, then add, subtract, multiply, and divide integers in the context of real-world problems. Given a real-world context, students simplify an expression using four integer operations and the order of operations.

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## SUGGESTED MATERIALS FOR THIS DOMAIN

6	7	
✓	✓	Common objects such as tennis shoes, cereal boxes, etc.
	✓	Copies of restaurant menus
✓		Counters (two-color, chips, etc.)
✓	✓	Graph paper
✓		Newspapers or grocery ads
✓		Percent wheel

## KEY VOCABULARY

6	7	
✓	✓	<b>commission</b> a percentage of sales paid to a salesperson
✓		<b>complex fraction</b> a fraction with a fraction in the numerator and/or a fraction in the denominator
✓	✓	<b>constant of proportionality</b> same as unit rate
	✓	<b>coordinate plane</b> a plane formed by the intersection of a horizontal number line (called the $x$ -axis) with a vertical number line (called the $y$ -axis). The number lines intersect at their zero points, called the origin
	✓	<b>covariance</b> a measurement of how related the variances are between two variables. The extent to which any two random variables change together or vary together.
✓	✓	<b>discount</b> amount a store takes off of the original price of an item. It is usually expressed as a percent or fraction.
✓		<b>double number lines</b> two number lines used when quantities have different units to easily see there are numerous pairs of numbers in the same ratio
✓	✓	<b>equation</b> a mathematical statement of the equality of two mathematical expressions. An equation uses a sign stating two things are equal ( $=$ ).
✓		<b>equivalent ratios</b> ratios that have the same value
✓		<b>gratuity</b> tip
✓	✓	<b>markdown</b> a reduction in price
✓	✓	<b>markup</b> the difference between the wholesale price and the selling price
✓	✓	<b>origin</b> on a coordinate plane, the point $(0, 0)$
✓	✓	<b>percent</b> a ratio per 100 such as 25% is 25 parts of 100

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## KEY VOCABULARY

6 7

✓	✓	<b>percent error</b> the ratio of the error compared to the exact value. For example, my estimate was off by 7. The exact value was 35, so the percent error is $\frac{7}{35} \times 100\% = 20\%$ .
✓	✓	<b>percent increase/decrease</b> the amount of increase or decrease expressed as a percent of the original amount
✓	✓	<b>proportion</b> two equal ratios
✓	✓	<b>proportional reasoning</b> multiplicative reasoning as opposed to additive reasoning. It is often used when finding the better buy, sharing two items with three students, adjusting calculations of travel time with different speeds, or calculating a sale price when everything is 40% off.
✓		<b>rate</b> ratio that compares two quantities of different units such as 3 ft per second
✓	✓	<b>ratio</b> comparison of two quantities
✓		<b>ratio language</b> language used to describe a ratio relationship in number or quantity between two things such as <i>"For every vote candidate A received, candidate C received nearly three votes"</i>
	✓	<b>ratio table</b> a table that shows the relationships between different ratios and/or a comparison of two or more quantities
✓	✓	<b>simple interest</b> the formula is $I = prt$ , where $I$ is interest, $p$ is principle, $r$ is rate, and $t$ is time
✓		<b>simplify a ratio</b> divide each number in the ratio by its greatest common factor; $\frac{2}{6}$ simplifies to $\frac{1}{3}$ by dividing 2 and 6 by 2
✓		<b>tape diagram</b> a drawing that looks like a segment of tape, used to illustrate number relationships; Also known as a strip diagram, bar model, fraction strip, or length model
✓	✓	<b>unit rate</b> ratio comparing an amount to one



# Ratios and Proportional Relationships

## 6.RP.A\*

## Cluster A

*Understand ratio concepts and use ratio reasoning to solve problems.*

### STANDARD 1

**6.RP.A.1:** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

### STANDARD 2

**6.RP.A.2:** Understand the concept of a unit rate  $a/b$  associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. *For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”<sup>1</sup>*

<sup>1</sup>Expectations for unit rates in this grade are limited to non-complex fractions.

### STANDARD 3

**6.RP.A.3:** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity); solve problems involving finding the whole, given a part and the percent.
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

\*Major cluster

## Ratios and Proportional Relationships 6.RP.A

**Cluster A: Understand ratio concepts and use ratio reasoning to solve problems.**

### Grade 6 Overview

The focus for this cluster is the study of ratio concepts and the use of proportional reasoning to solve problems. Students learn how ratios and rates are used to compare two quantities or values and how to model and represent them. Sixth graders find out how ratios are used in real-world situations and discover solutions to percent problems using ratio tables, tape diagrams, and double number lines. Students also convert between standard units of measure.

### Standards for Mathematical Practice

**SFMP 1. Make sense of problems and persevere in solving them.**

Sixth graders interpret and solve ratio problems.



## STANDARD 1 (6.RP.A.1)

*Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”*

In this standard, students learn to compare two quantities or measures such as 6:1 or 10:2. These comparisons are called ratios. Students discover that ratios can be written and described in different ways. For instance, 6:1 uses a colon to separate values.

Ratios can also be stated with words such as 6 to 1, or as a fraction such as  $\frac{6}{1}$ . Standard 1 focuses on understanding the concept of a ratio, however, students should use ratio language to describe real-world experiences and use their understanding for decision making.

### What the TEACHER does:

- Help students discover that a ratio is a relationship or comparison of two quantities or measures. Ratios compare two measures of the same types of things such as the number of one color of socks to another color of socks or two different things such as the number of squirrels to birds in the park. Ratios compare parts to a whole (part:whole) such as 10 of our 25 students take music lessons. Ratios can also compare a part of one whole to another part of the same whole (part:part) such as the ratio of white socks in the drawer to black socks in the drawer is 4:6. Ratios are expressed or written as  $a$  to  $b$ ,  $a:b$ , or  $\frac{a}{b}$ .
- Compare and model ratios with real-world things such as pants to shirts or hot dogs to buns. Ratios can be stated as the comparison of 10 pairs of pants to 18 shirts and can be written as  $\frac{10}{18}$ , 10 to 18, or 10:18 and simplified to,  $\frac{5}{9}$ , 5 to 9, or 5:9. Ensure that students understand how the simplified values relate to the original numbers.
- Ask students to create or find simple real-world problems to use in their learning such as, “There are 2 Thoroughbred horses and 6 Appaloosa horses in the field. As a ratio of Thoroughbreds to Appaloosas it is:  $\frac{2}{6}$  or 2 to 6 or 2:6 or simplified as  $\frac{1}{3}$ , 1 to 3, or 1:3. Or, there are 14 girls and 18 boys in our math class. As a ratio of girls to boys it is:  $\frac{14}{18}$ , 14 to 18, or 14:18 or simplified as  $\frac{7}{9}$ , 7 to 9, or 7:9.” Invite students to share their real-world examples of ratios and use ratio language to describe their findings such as, “for every vote candidate A received, candidate C received nearly three votes.” The problems students select or write can also be used as cyclical reviews with distributed practice throughout the school year.
- Focus on the following vocabulary terms: *ratio*, *compare*, and *simplify*.

### What the STUDENTS do:

- Understand that a ratio is a comparison between quantities.
- Determine when a ratio is describing part-to-part or part-to-whole comparison.
- Describe ratio relationships between two quantities using ratio language.
- Use the different ratio formats interchangeably (4:5, 4 to 5,  $\frac{4}{5}$ ).

### Addressing Student Misconceptions and Common Errors

Some sixth graders may confuse the order of the quantities such as when asked to write the ratio of boys to girls in the sentence, “There are 14 girls and 18 boys in our math class.” Instead of writing 18:14, some students may write 14:18. Other students may not recognize the difference between a part-to-part ratio and a part-to-whole ratio such as, “There are 14 girls compared to 18 boys in the class (14:18 part-to-part); however, 14 of the 32 students in our class are girls (14:32 part-to-whole).” To address these common misconceptions, ask students to label the quantities they are comparing such as 14 girls/18 boys.

## STANDARD 2 (6.RP.A.2)

Understand the concept of a unit rate *alb* associated with a ratio  $a:b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is  $\frac{3}{4}$  cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”<sup>1</sup>

<sup>1</sup>Expectations for unit rates in this grade are limited to non-complex fractions.

This standard focuses student learning on the concept of a unit rate as a special kind of ratio. Students compare different units of measure such as the amount of money earned to the hours worked while babysitting and calculate unit rates by setting up ratios and simplifying them. Students understand a situation in ratio form and write the unit that describes the situation using appropriate rate language with words such as *per* and symbols such as / to compare different units or measures.

### What the TEACHER does:

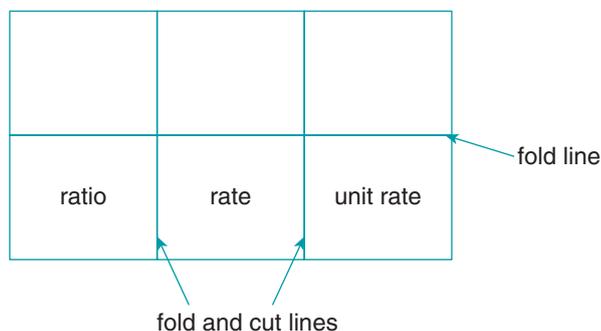
- Begin by exploring the difference between a ratio and a rate. Rate is a special ratio that compares two quantities with different units of measure. Share multiple examples for students to make sense of the concept for rate such as, “LaShanda babysat for \$35 for 7 hours.” Or, “Dad’s new truck got 400 miles on 20 gallons of gas.” Then explore the unit rate that expresses a ratio as part-to-one. Generate examples such as “LaShanda is paid a unit rate of \$5 per 1 hour for babysitting (5:1)” and “My dad’s new truck gets 20 miles per gallon of gas (20:1).”
- Ask students to locate and share real-world examples of cost per item or distance per time in newspapers, ads, or other media. (Note that in sixth grade, students do not work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.)
- Model how to convert rates from fraction form to word form using *per*, *each*, or *@* such as 360 miles/12 gallons of gas = 30 miles per gallon of gas. Allow students to talk with each other and their teacher to make sense of what they are learning and then write and share several rate conversion examples of their own.
- Focus on the following vocabulary terms: *ratios*, *rates*, *unit rates*, *compare*, and *per/@*. Math journals or exit slips at the end of a math class with writing prompts such as, “An example of a ratio and a problem that goes with it is. . . .” provide closure.
- Provide cyclical, distributed practice over time to continually review simple unit rate problems.

### What the STUDENTS do:

- Understand rate as a ratio that compares two quantities with different units of measure.
- Understand that unit rates are the ratio of two measurements or quantities in which the second term means “one” such as 60 miles per one hour.
- Interpret rate language with the @ symbol or with the words *per* and/or *each*.
- Solve unit rate problems.

### Addressing Student Misconceptions and Common Errors

Students often confuse the terms *ratio*, *rate*, and *unit rate*. Try using a paper foldable with vocabulary definitions to help students with these confusing terms. To make the foldable, divide an  $8\frac{1}{2} \times 11$ -inch sheet of blank paper in half horizontally. Then fold it into thirds as if a letter is being folded to fit an envelope. Unfold and write a term on each of the sections. On the inside of the foldable, write the definitions that match each term. Students may want to cut on the vertical fold lines to flip up each section to practice the definitions.



## STANDARD 3 (6.RP.A.3)

Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Find a **percent** of a quantity as a rate per 100 (e.g., 30% of a quantity means  $\frac{30}{100}$  times the quantity); solve problems involving finding the whole, given a part and the percent.
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

In these standards, students use reasoning about multiplication and division to solve a variety of ratio and rate problems about quantities. They make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. They use tables to compare ratios and solve unit rate and constant speed problems. Problems involving finding the whole given a part and the percent, such as 20% of a quantity means  $\frac{20}{100}$ , are also a focus. For these standards, students can use equivalent ratio tables, tape diagrams, double number lines, or equations. Students connect ratios and fractions.

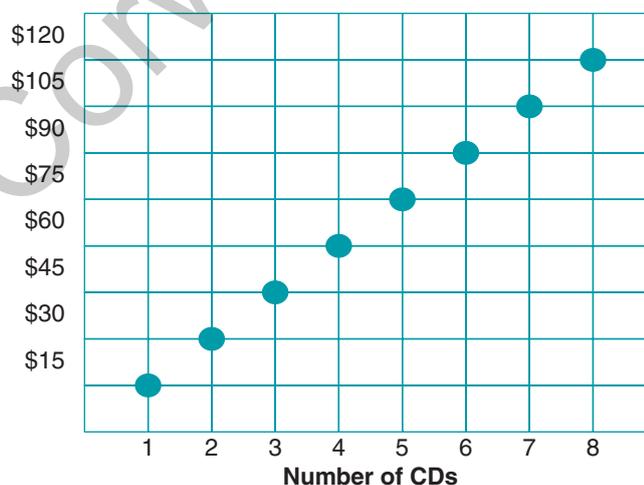
### What the TEACHER does:

- Explore ratios and rates used in ratio tables and graphs to solve problems. Pose a ratio situation problem with students such as “3 CDs cost \$45. What would 8 CDs cost? How many CDs can be purchased for \$150.00?” To solve the problem, students can use ratios, unit rates, and multiplicative reasoning by creating and filling in the missing values on a chart. They should note that if three CDs cost \$45, one CD will cost \$15. Every CD purchased is an additional \$15. \$15 times the number of CDs = the cost. They write an equation such as  $C = \$15n$ .

# of CDs	Cost
3	\$45
8	??

- Ask students to plot the points on a coordinate plane and draw conclusions about what is happening with the problem above. Students should reason that for every one movement to the right on the x-axis, the y-axis increases to 15x. Also, for every one movement to the left on the x-axis, the y-axis decreases by 15.

CD Costs



- Investigate unit rate problems, including unit pricing such as, “Quick Stop has 12-oz. drinks for \$.99. Stop Here has 16-oz. drinks for \$1.19. Which drink costs the least per ounce?” Assign students to create ratio and rate reasoning examples to compare and solve real-world problems. Students could use newspapers, store ads, or online ads to find the examples and make the comparisons. Ask students to use reasoning to determine the better buys.
- Explore finding a percent of a quantity as a rate per 100 such as 40% of a quantity means  $\frac{40}{100}$  times the quantity. Noting that a percent is a rate per 100, model how a

(continued)

**What the TEACHER does (continued):**

percent can be represented with a hundreds grid by coloring in 40 units. Have students write this as a fraction ( $\frac{40}{100}$ ), a decimal (0.40), and a percent (40%). Consider using a percent wheel (see Reproducible 1) or use double number lines and tape diagrams in which the whole is 100 to find the rate per hundred.

- Solve problems involving finding the whole, given a part and the percent such as, “What is 40% of 60? 80% of what number is 300? Or 50 is 30% of what number?”
- Examine the process of how to use ratio reasoning to convert measurement units such as, “How many centimeters are in 5 feet?” Use the information that 1 inch  $\approx$  2.54 cm. Represent the conversion of 12 inches = 1 ft as a conversion factor in ratio form,  $\frac{12 \text{ inches}}{1 \text{ foot}}$ .

$$\text{Then multiply } \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{5 \text{ ft}}{1} = 60 \text{ inches.}$$

$$\text{Then } 60 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 152.4 \text{ cm.}$$

(Note that conversions can be made between units within a measurement system such as inches to feet or between systems such as miles to centimeters.)

- Allow students to talk with each other and their teacher to make sense of what they are learning.
- Focus on the following vocabulary terms: *ratios, rates, unit rates, equivalent ratios, percents, ratio tables, and tape diagrams.*
- Provide cyclical, distributed practice over time to continually practice unit rate problems.

**What the STUDENTS do:**

- Create and interpret a table of equivalent ratios.
- Plot pairs of values from a table to a coordinate plane.
- Use a table to compare ratios and find missing values using ratios.
- Explain the difference between a ratio and a unit rate.
- Understand that rate problems compare two different units, such as revolutions per minute.
- Solve real-world problems using ratios and rates.
- Reason to determine the better buy.
- Write a percent as a rate over 100, including percents greater than 100 and less than 1.
- Find the percent of a number using rate methods.
- Represent the relationship of part to whole to describe percents using models.
- Convert units by multiplication or division.

**Addressing Student Misconceptions and Common Errors**

Some sixth graders misunderstand and believe that a percent is always a natural number less than or equal to 100. To help with this misconception, provide examples of percent amounts that are greater than 100% and percent amounts that are less than 1%. Try using a percent wheel for developing this understanding. See Reproducible 1.

Notes

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**Standard: 6.RP.A.1.** Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

**Mathematical Practice or Process Standards:**

**SFMP 2. Reason abstractly and quantitatively.**

Students solve problems by analyzing and comparing ratios and unit rates in tables, equations, and graphs.

**SFMP 4. Model with mathematics.**

Students model real-life situations with mathematics and model ratio problem situations symbolically.

**SFMP 6. Attend to precision.**

Students communicate precisely with others and use clear mathematical language when describing a ratio relationship between quantities.

**Goal:**

Students use real-world objects to compare two quantities such as the number of red candy pieces to the number of green candy pieces in the same bag (part to part) and the number of parts to a whole such as the number of red candy pieces compared to the total number of candy pieces in the entire bag (part to whole).

**Planning:**

**Materials:** plastic bags with approximately 35–40 pieces of M & M’s™ candies or 1-inch color tiles, paper and pencil to write the ratios

**Sample Activity**

- Give each student a bag of M & M’s™ to compare and model ratios. Divide students into partner pairs. Ask them to write a ratio comparing the number of M & M’s™ they have to the number their partner has. Students will count and compare the number of M & M’s™ each have in their own bags and then write the comparison such as  $\frac{36}{40}$  or 36:40. Facilitate a discussion about how they just compared a whole to a whole.
- Next, ask students to compare the number of red M & M’s™ in their bags to the number of brown M & M’s™. Students will count and record comparisons such as 8 red/14 brown or 8:14. Facilitate a discussion about how this ratio compares a part of one whole to another part of the same whole (part to part). Ask students to create their own part to part ratios with their M & M’s™ and record.
- Ask students to count the number of yellow M & M’s™ and compare that number to the entire number of M & M’s™ in the bag, such as 7 yellow compared to all 36 in the bag. Have students record the ratio such as  $\frac{7 \text{ yellow}}{36 \text{ bag}}$  or 7:36. Facilitate a discussion leading students to reason that the ratio they just created is a part:whole ratio. This can be done by reviewing the other types of ratios created earlier in this lesson.



**PLANNING PAGE**

**Standard:**

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**Mathematical Practice or Process Standards:**

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**Goal:**

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**Planning:**

*Materials:*

*Sample Activity:*

**Questions/Prompts:**

**Differentiating Instruction:**

*Struggling Students:*

*Extension:*

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# Ratios and Proportional Relationships

## 7.RP.A\*

### Cluster A

Analyze proportional relationships and use them to solve real-world and mathematical problems.

#### STANDARD 1

**7.RP.A.1:** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{\frac{1}{2}}{\frac{1}{4}}$  miles per hour, equivalently 2 miles per hour.*

#### STANDARD 2

**7.RP.A.2:** Recognize and represent proportional relationships between quantities.

- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Represent proportional relationships by equations. *For example, if total cost  $t$  is proportional to the number  $n$  of items purchased at a constant price  $p$ , the relationship between the total cost and the number of items can be expressed as  $t = pn$ .*
- Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

#### STANDARD 3

**7.RP.A.3:** Use proportional relationships to solve multi-step ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

\*Major cluster

## Ratios and Proportional Relationships 7.RP.A

**Cluster A: Analyze proportional relationships and use them to solve real-world and mathematical problems.**

### Grade 7 Overview

These standards extend what students learned in Grade 6 about ratios to analyzing proportions and proportional relationships. Students calculate unit rates with complex fractions and move to recognizing and representing proportional relationships in equations and on graphs. These skills and understandings are used to solve multi-step ratio and percent problems involving real-world scenarios such as interest, tax, shopping sales, and so on.

### Standards for Mathematical Practice

#### SFMP 1. Make sense of problems and persevere in solving them.

Students solve multi-step ratio and real-world percent problems.

#### SFMP 3. Construct viable arguments and critique the reasoning of others.

Students recognize proportional relationships from non-proportional ones and discuss their reasoning with others.



## STANDARD 1 (7.RP.A.1)

Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{\frac{1}{2}}{\frac{1}{4}}$  miles per hour, equivalently 2 miles per hour.

This standard focuses on computing unit rates using ratios of fractions known as complex fractions. In a complex fraction, the numerator, denominator, or both are fractions. In the standard,  $\frac{\frac{1}{2}}{\frac{1}{4}}$  is an example of a complex fraction. Complex fractions can be interpreted as division statements. For example,  $\frac{\frac{1}{2}}{\frac{1}{4}}$  can be thought of as  $\frac{1}{2} \div \frac{1}{4}$ . Applications include situations where the quantities are measured in different units such as miles per hour, pounds per square foot, feet per second, and so on.

### What the TEACHER does:

- Explore unit rates with ratios of fractions and compare them to unit rates with whole numbers from Grade 6.
- Treat complex fractions as division of fractions.
- Set up error analysis scenarios where students can identify errors in computing unit rates with complex fractions. For example, Homer calculated that if a person walks  $\frac{1}{2}$  mile every  $\frac{1}{4}$  hour, the unit rate is 2 miles. However, Homer made an error. Find his error, correct it, and explain to Homer why 2 miles is not the correct answer.
- Provide opportunities for students to compute the unit rates in real-world problems.

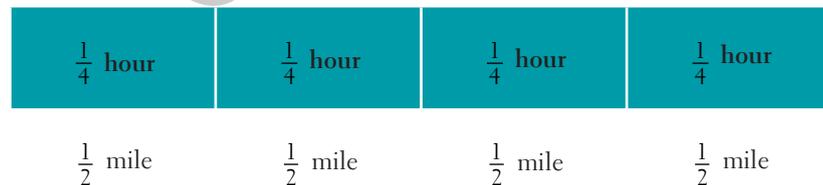
### What the STUDENTS do:

- Discover that the structure of computing unit rates with whole numbers is the same concept as unit rates with ratios of fractions.
- Compute unit rates in real-world problems that involve complex fractions.
- In writing, explain the errors that can be made when computing unit rates with complex fractions and unlike units.

### Addressing Student Misconceptions and Common Errors

It is not uncommon to find seventh-grade students who are not fluent with fraction division. Sometimes the format of a complex fraction confuses them when they are used to seeing fraction division written horizontally as  $\frac{1}{2} \div \frac{1}{4}$ . Discuss how the division bar in the complex fraction means the same as the symbol  $\div$ .

For students having difficulty understanding unit rate and those having trouble with different units such as miles per hour, pictures and diagrams may help. Use the example from this standard: *If a person walks  $\frac{1}{2}$  mile in each  $\frac{1}{4}$  hour, compute the unit rate as the complex fraction  $\frac{\frac{1}{2}}{\frac{1}{4}}$  miles per hour, equivalently 2 miles per hour.* Model with a diagram as shown. The bar represents 1 hour broken into  $\frac{1}{4}$  hour segments.



From this diagram, students can see that the word problem is showing  $\frac{1}{2}$  mile every  $\frac{1}{4}$  hour.

## STANDARD 2 (7.RP.A.2)

Recognize and represent proportional relationships between quantities.

- a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Sections a–d of 7.RP.A.2 break down the standard to give guidance on ways to recognize and represent proportional relationships.

This standard emphasizes two methods for deciding whether a proportional relationship exists. One method is to use equivalent ratios in a table. If the ratios are equivalent, then you have a proportional relationship such as:

# of people in a room	1	2	3	4	5
# of hands in the room	2	4	6	8	?

The other method is to graph the relationship on a coordinate plane and observe whether the graph is a straight line that goes through the origin. Note that computation using cross-multiplication is not a part of this standard.

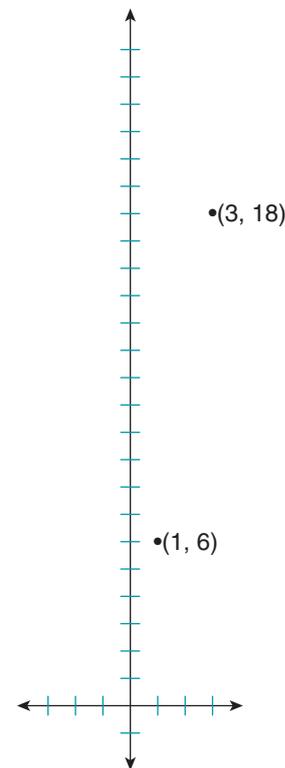
### What the TEACHER does:

- Explore proportional reasoning scenarios with students to be sure they understand the meaning of proportional relationships in context before using the tables or graphs. While some number combinations may be proportional, the real-world example attached to the numbers may not be. Use examples and non-examples for students to identify and compare. An example is: “2 music downloads cost \$1.98; therefore, 4 music downloads cost \$3.96.” A non-example is: “Three boys can run a mile in 10 minutes; therefore, 6 boys can run a mile in 20 minutes.”
- Ask students to write their own examples and non-examples of proportional relationships. Student work can be shared and discussed.
- Discuss equivalent ratios with the students. Ask them to suggest some equivalent pairs. Relate to equivalent fractions. Display the pairs as the students suggest them in the form  $\frac{a}{b} = \frac{c}{d}$ , where  $b \neq 0$  and  $d \neq 0$ . Define two equivalent ratios as a proportion.
- Pose examples of proportions written with the quantities in different positions. Encourage students to decide if there is more than one correct way to set up a proportion. For example: “Set up a proportion showing that 3 out of 15 students are girls is the same ratio as 1 out of 5 students are girls.”

$$\frac{3}{15} = \frac{1}{5} \text{ or } \frac{15}{3} = \frac{5}{1} \text{ or } \frac{1}{3} = \frac{5}{15} \text{ or } \frac{3}{1} = \frac{15}{5}$$

Ask students to explain how they know  $\frac{15}{1} = \frac{5}{3}$  is not a correct proportion for the example.

- Provide examples of equivalent and non-equivalent ratios to students for them to test with a table to decide if they are proportions. Conversely, present students with a table for a context and ask them to determine if all of the entries in the table are proportional.
- Graph two ratios on a coordinate plane from a proportional scenario and look for a straight line that goes through the origin to determine if the two ratios are proportional. For example: “Maria sells necklaces and makes a profit of \$6 for each necklace. How much money does she make for selling 3 necklaces?”
- Pose the task to students: Select other points on the graphed line and determine if they are also proportional.



### What the STUDENTS do:

- Sort real-world examples of proportional relationships from non-examples. Students can create their own examples to demonstrate that they understand the concept of proportional relationships when there is a context attached.
- Communicate orally and/or in writing that a proportion is a statement of two equivalent ratios. Students apply what they know about equivalent fractions to equivalent ratios.
- Model proportional relationships by creating tables; determine if a proportional relationship exists from a given table.
- Model relationships on graphs to determine if they are proportional.
- Test their hypotheses about whether a proportional relationship exists between any two points on the lines graphed. Students may draw the conclusion that all points on the line are proportional to all other points on the line by relying on tables, verbal statements, or logical arguments to draw the conclusion.

### Addressing Student Misconceptions and Common Errors

While graphing, students may need to be reminded that the same types of quantities need to be graphed on the same axis. For example, when checking to determine if 10 cans of soda for \$2 is proportional to 50 cans of soda for \$10, the cans of soda must both be represented on the same axis and the dollar amounts must be on the other axis. Ensure students are using graph paper or graphing calculators for all graphing. Remind them to label the axes.

*Notes*

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**b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.**

This standard focuses on proportional relationships that can be represented as tables, graphs, equations, diagrams, and verbal descriptions. Students have already seen tables, graphs, and verbal descriptions. The unit rate on a graph is the point where  $x = 1$ . In an equation, it is the slope represented by the coefficient,  $m$ , in the formula  $y = mx + b$ . The terms *unit rate*, *constant of proportionality*, and *slope* are equivalent. Note that students are only required to read and interpret equations in this standard.

**What the TEACHER does:**

- Facilitate a discussion about representations of proportional relationships using a real-world scenario. For example, beginning with the verbal description: Mark was looking to fertilize his lawn, which is 432 sq. ft. He read the packages of 5 different fertilizer bags to see how much should be used. Bag A stated 2 ounces per 4 square feet, Bag B stated 4 ounces per 8 square feet, Bag C stated 1.5 ounces per 3 square feet, and Bag D stated 6 ounces per 12 square feet. Are these rates proportional? If yes, what is the unit rate? How much fertilizer does Mark need for his lawn?
- Using a real-world context have students determine if the relationship is proportional using graphs and/or tables. If it is proportional, facilitate a discussion with the class on the unit rate.
- Share a verbal description of a proportional relationship and ask students to interpret it with a diagram such as bars. Encourage students to write how they interpreted the proportional relationship.
- Introduce equations as a statement of the proportional relationship. For the fertilizer story the equation is  $f = 2z$ , where  $f$  is the amount of fertilizer needed and  $z$  is the size of the lawn in square feet.
- Provide students with a real-world proportional relationship expressed in a verbal description, graph, table, and equation. Challenge students to work with a partner to compare how the unit rate is expressed in each representation. Share student discoveries in a large class discussion.

**What the STUDENTS do:**

- Model proportional relationships several different ways.
- Translate a proportional relationship from a verbal description into a diagram and explain in writing how the diagram shows a proportional relationship.
- Determine the unit rate from equations, graphs, tables, diagrams, and verbal descriptions of proportional relationships.
- Discover that the unit rate (constant of proportionality) is the numerical coefficient in the equation of a proportional relationship.

**Addressing Student Misconceptions and Common Errors**

Finding the unit rate from a graph can be confusing. Some students cannot remember if the unit rate is the  $(1, y)$  or  $(x, 1)$  point. It is helpful to have a familiar unit rate students can recite such as 1 CD for \$11.99 that helps them remember the  $x$ , which is first in a coordinate pair, is the 1 and the  $y$  is the unit rate.

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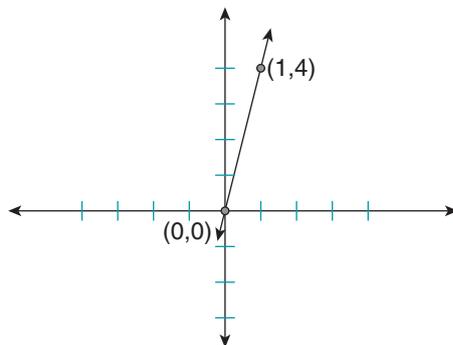


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d. Explain what a point  $(x, y)$  on the graph of a proportional relationship means in terms of the situation, with special attention to the points  $(0, 0)$  and  $(1, r)$  where  $r$  is the unit rate.

An example of a proportional situation is: The scale on a map suggests that 1 centimeter represents an actual distance of 4 kilometers. The map distance between two towns is 8 centimeters. What is the actual distance? The graph of this relationship is represented as:



Note the points  $(0,0)$  and  $(1,4)$ . The 4 is the unit rate or slope of the line for the equation  $d = 4c$ , where  $d$  is total distance and  $c$  is the number of centimeters.

#### What the TEACHER does:

- Present students with a verbal description of a proportional relationship and build the graphical representation with the students. Be sure students give input on the labels for the  $x$ - and  $y$ -axes. Facilitate a discussion about the graph with students asking them for the meaning of individual points and asking students to justify their responses.
- Have students compare graphs that show proportional relationships and talk to a partner about what they notice.
- Focus on points  $(0, 0)$  and  $(1, r)$ , the origin and the unit rate, respectively.
- Use points that are not whole numbers and points where students need to estimate the coordinates.

#### What the STUDENTS do:

- Explain the meaning of a point on a graph in the context of the situation. Students should be able to explain examples with words such as, “Point  $(5, 7)$  is the point that represents 5 health bars for \$7.00” or “ $(1, 10)$  represents the unit rate (constant of proportionality), meaning 1 teacher for every 10 students at the school.”
- Discover that graphed proportional relationships are straight lines.

#### Addressing Student Misconceptions and Common Errors

When finding points on a line that represents a real-world proportional relationship, students may think that the line stops at the origin. The teacher should show that the line continues into Quadrant III but that the points are not appropriate for the real-world situation. For example, in a proportional relationship between the number of teachers and the number of students at a grade level, it does not make sense to have  $-3$  teachers.

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## STANDARD 3 (7.RP.A.3)

Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

In this standard students solve problems involving proportional relationships. Students set up and solve proportions using cross-multiplication. For example: “Directions to make a tablecloth call for  $\frac{3}{4}$  yard of ribbon for every 2 yards of fabric. If you increase the amount of fabric used to 3 yards, how much ribbon will be needed?” The proportion is  $\frac{\frac{3}{4}}{2} = \frac{x}{3}$ . To cross-multiply:

$$3 \cdot \frac{3}{4} = 2x$$

Problems for this standard should be multi-step and include contexts with simple interest, tax, tips, commissions, percent error, percent increase/decrease, discounts, fees, markups, markdowns, discount, sales, and/or original prices.

To calculate a percent increase from 2 to 10, find the difference between the two numbers, in this case,  $10 - 2 = 8$ . Take the difference, 8, and divide by the original number:  $\frac{8}{2} = 4$ . Multiply the quotient by 100:  $4 \times 100 = 400\%$ .

### What the TEACHER does:

- Focus time on the vocabulary for this standard. Paper foldables, word walls, graphic organizers, using words in context, and writing stories all give students a chance to clarify the meaning of these terms, which they may encounter in daily life but not fully understand. Bring in items familiar to students such as tennis shoes, a six-pack of soda, and so on and use them to model situations that use the vocabulary. Vocabulary should include simple interest, tax, tip/gratuity, discount, commission, fees, sale, markup, markdown, and original price.
- Use cross-multiplication to solve problems involving proportional relationships. Use numbers in your problems that do not lend themselves easily to mental arithmetic.
- Begin with single-step problems and move to multi-step using a wide variety of contexts. Make use of everyday examples such as finding sales online, in print media, and on TV.
- Ask students to write problems that can be solved with setting up proportions prompted by media ads.
- Introduce students to percent increase/decrease and percent error problems. Encourage students, through questioning, to discover the similarities among the formulas for these concepts.

### What the STUDENTS do:

- Explore use of the vocabulary words in this standard by finding examples in the media and explain how they are used in each situation.
- Solve problems involving proportions using cross-multiplication.
- Solve problems involving percent error and percent increase/decrease.
- Use the structure of percent error and percent increase/decrease problems to explain how the formulas for these concepts are similar.

### Addressing Student Misconceptions and Common Errors

Students may have misconceptions about the vocabulary commonly used in the media such as sale, discount, and tax. It is important to discuss what students already know about these words in order to correct any pre-existing misconceptions. For individuals with difficulties with particular words, use graphic organizers such as the Frayer model (see Reproducible 2). Acting out situations can help students remember certain steps. For example, acting out shopping for a pair of tennis shoes and a tennis racket and paying tax at the register will help students remember that tax is calculated on the cost of the total bill where the items bought need to be added up before tax is calculated.

## Sample PLANNING PAGE

**Standard: 7.RP.A.3.** Use proportional relationships to solve multi-step ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

### Mathematical Practice or Process Standards:

**SFMP 1. Make sense of problems and persevere in solving them.**

Students solve multi-step problems involving percents.

**SFMP 6. Attend to precision.**

Students check answers to see if they are reasonable.

### Goal:

Students demonstrate understanding of the vocabulary tax and tip (gratuity) while solving multi-step problems.

### Planning:

**Materials:** paper and pencil, copies of restaurant menus

**Sample Activity:**

- Provide groups of students with a menu and the following problem:

#### Route 15 Lunch Market Take-Out Menu

Pulled Pork Sandwich	\$4.49
Hamburger	\$3.39
Cheeseburger	\$3.99
Chicken Sandwich	\$4.49
Fish Sandwich	\$4.49
Hot Dog	\$1.99

Add fries and a 12-oz.  
drink for \$1.99 more!



Image courtesy of clipart.com.

Your group decides to keep working through lunch today so you will order lunch from the Route 15 Lunch Market.

Use the take-out menu to figure out:

- a. What you want to order
- b. How much the food will cost
- c. How much tax will be added to the bill at a rate of 5.5%
- d. What percent tip you will leave and then calculate the tip
- e. Your total cost for lunch, including tax and tip

Remember to write everything down clearly, step by step, so that the waitress at the restaurant can fill your order accurately.

**Questions/Prompts:**

- Are students overwhelmed by the number of steps? Suggest they make a list of each part of the problem.
- Are students confused about whether they compute tax or tip first? Ask questions for students to reason if it makes sense to pay a tip on the tax.
- Is the order illegible? Ask students to read what they wrote to you. Point out that if you cannot read it, neither can the workers at the restaurant. Suggest ways to improve the clarity. Provide graph paper for groups that may need it to keep numbers lined up.
- Ask, “Does your answer make sense?”

**Differentiating Instruction:**

**Struggling Students:** For students struggling with the number of steps, break the problem down into steps. Write each step on an index card. Ask the students to order the steps so they can discuss what should come first before they tackle the problem.

**Extension:** Ask students to create their own word problems from the menu for their classmates.

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**PLANNING PAGE**

**Standard:**

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**Mathematical Practice or Process Standards:**

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**Goal:**

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**Planning:**

*Materials:*

*Sample Activity:*

**Questions/Prompts:**

**Differentiating Instruction:**

*Struggling Students:*

*Extension:*

## Reflection Questions: Ratios and Proportional Relationships

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1. The domain *Ratios and Proportional Relationships* appears in Grades 6 and 7 only. What ideas from Grades K–5 prepare students for the study of ratios and proportional reasoning?

2. This domain has several big ideas. Select one main idea and trace its development through Grades 6–7.

3. What big ideas from this domain are foundational for the functions domain in Grade 8?

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