

Introduction

A Brief History of the Common Core

Contrary to popular belief, academic standards are not new. In fact they have been around for more than twenty-five years. The first set of curriculum specific standards, *The Curriculum and Evaluation Standards for School Mathematics*, was released by the National Council of Teachers of Mathematics (NCTM) in 1989, followed by an updated set of standards, *Principles and Standards for School Mathematics*, in 2001. Both of these documents provided a vision for K–12 mathematics by grade level band. They also formed the foundation for most states’ grade level standards.

In April 2009, the National Governors Association and the Council of Chief State School Officers met to discuss the creation of the Common Core State Standards Initiative. The purpose of such an initiative was to develop a set of common standards across states in order to balance the quality of mathematics instruction and learning. Following that meeting, the process of writing the common core standards began. The standards development team, led by William McCallum, Phil Daro, and Jason Zimba, included mathematicians, mathematics educators, mathematics education researchers, and classroom teachers. The process included open invitation for feedback, not only from mathematics educators and associations including NCTM but also from the general public. This feedback was considered, and much of it was incorporated into the final document, which was released in June 2010. Following the release of the standards, individual states went through their own processes for reviewing, adopting, and, if necessary, ratifying the adoption of the Common Core State Standards.

The Common Core State Standards for Mathematics

“The Common Core State Standards are a clear set of shared goals and expectations for the knowledge and skills students need in English language arts and mathematics at each grade level so they can be prepared to succeed in college, career, and life.” (www.corestandards.org/about-the-standards/frequently-asked-questions/#faq-2303)

The Common Core State Standards for Mathematics (CCSSM) includes two critical components of learning mathematics. The content standards explicitly outline the mathematics we want students to know and be able to do at each grade level. The content standards of the common core are fewer in number than most previous state standards; at the same time, the expectation is that students will develop a deeper understanding of that content so less time is spent on reteaching from year to year. Additionally, the standards were carefully constructed to show connections among ideas at a grade level as well as vertical progressions across grades. For example, you will find that the standards in Grade 1 develop from the mathematical work that students have completed in kindergarten. Similarly, the standards in Grade 2 develop from work completed in Grade 1. Thus, it is important for teachers to be knowledgeable of the standards not only at the level they are teaching but also at the preceding grade level and the next grade level.

The second group of standards, the Standards for Mathematical Practice, describes the habits of mind students should develop as they do mathematics. These eight standards are the same across all grade levels, K–12. As teachers plan mathematics lessons, they should consider how students will use the practices in learning and doing mathematics.

The Common Core Standards *are not* a curriculum. Decisions about mathematics programs, textbooks and materials, sequencing topics and units, and instructional frameworks are left for local or state school districts to make. They do not tell teachers how to teach. It is important to remember that they describe what students need to know and be able to do. Schools and teachers know best how to help students be successful with both the content and the practice standards.

The Common Core State Standards *do not* dictate specific assessments. Some states will be using assessments developed by *Partnership for Assessment of Readiness for College and Careers* (PARCC) or *Smarter Balanced Assessment Consortium* (SBAC, or Smarter Balanced). Others will develop and use their own assessments. Other facts and information can be found at <http://www.corestandards.org>.

Instructional Shifts

While the standards do not call for a particular instructional model or philosophy, they are based on the best of existing standards. What is different is that they call for specific instructional shifts: **focus**, **coherence**, and **rigor**.

Focus: The content standards call for greater focus on fewer topics. An examination of the mathematics standards of high-performing countries indicates that fewer, more focused topics at a grade level allow students to deepen their understanding of the mathematics and gain a stronger foundation for an ongoing study of mathematics. Within the standards, the major mathematical work of each grade level has been identified (www.corestandards.org). That means that not all of the content within a grade is emphasized equally in the standards. The list of content standards for a grade is not linear, nor is it a checklist. Some clusters require greater emphasis than others. They take more time for students to master with depth of understanding. The major work of K–2 includes addition and subtraction—concepts, skills, and problem solving; and place value. This means the majority of instructional time in K–2 (65% to 85%) should be spent on these mathematical topics. This does not mean that other standards should be skipped. Rather, the supporting standards should be taught to connect mathematical ideas to the essential standards. The additional standards provide students with experiences that will be foundational to work in future grades. Neglecting material will leave gaps in student skill and understanding.

Coherence: Many of us learned mathematics as a set of disconnected topics, with much of our skill based on tricks (“ours is not to reason why, just invert and multiply”) or mnemonic devices (Please Excuse My Dear Aunt Sally). In reality, mathematics is a coherent body of knowledge made up of topics that are connected and build on each other. The call for coherence in the content standards ensures that there are carefully constructed progressions from grade to grade so students build new understandings on the foundations laid in previous years. Each standard is not a new topic, but an extension of previous learning. In addition to the progressions across grade levels, the standards incorporate specific connections within a grade level. For example, as students learn the topics of addition and subtraction in the primary grades, the meaning of these operations is consistently connected to applying these understandings and skills to solving problems.

Rigor: The final instructional shift, rigor, refers to how to support students in developing deep understanding of each standard. Understanding does not develop by assigning more worksheets or more difficult examples and problems. Rather, it calls for instructional practice that balances conceptual understanding, procedural skills, and applying mathematical ideas to a variety of contexts.

The following descriptions of each component of rigor come from the standards document, which can be found at www.corestandards.org.

Conceptual understanding: The standards call for conceptual understanding of key concepts, such as place value. Students must be able to access concepts from a number of perspectives in order to see mathematics as more than a set of rules or procedures.

Procedural skills and fluency: The standards call for speed and accuracy in calculation. Students must practice core skills, such as basic facts and addition/subtraction computation, in order to have access to more complex concepts and procedures. Fluency is built upon conceptual understanding and, with young children, through the development of ideas through representations using concrete materials, pictures, numbers, and words.

Application: The standards call for students to use mathematics in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Major Work of K–2

To help drive the development of the content standards, between 65% and 85% of instructional time should focus on the major work for each grade level. (For K–2, the amount of time should be closer to 85%.) Areas of major work include:

Kindergarten—number ideas and counting, understanding addition as putting together and subtraction as taking from or taking apart, number facts with sums to 5, and work with numbers 11–19 to form the foundation of place value.

Grade 1—represent and solve problems with addition and subtraction, understand properties of addition and subtraction, fluently add and subtract within 10, work with sums to 20, extend the counting sequence to 120, understand foundations of place value using those ideas to add and subtract and measure lengths indirectly and by repeating length units.

Grade 2—represent and solve problems with addition and subtraction, fluently add and subtract within 20, understand place value, use place value and properties to add and subtract, measure and estimate lengths in standard units, and relate addition and subtraction to measuring.

Additional information on the focus for each grade level can be found in the resource section at the end of this book.

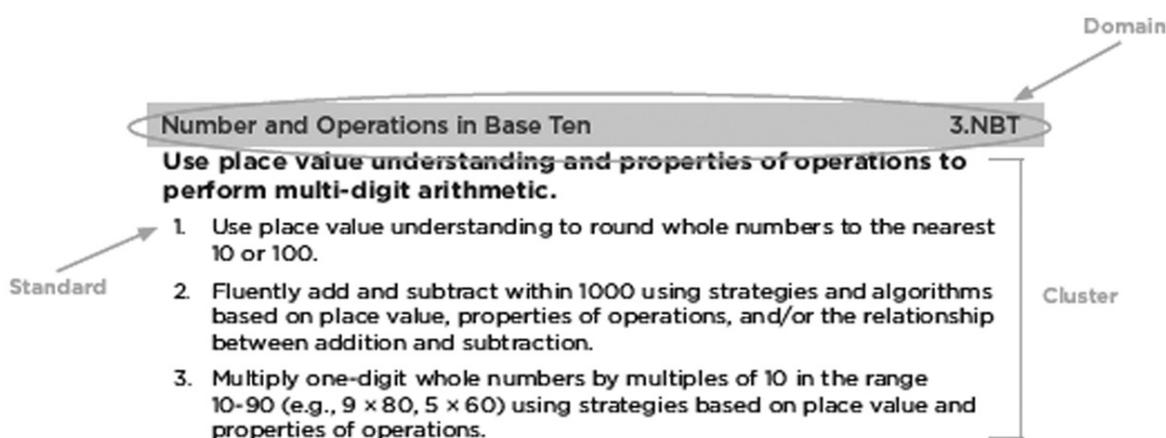
Common Core Word Wall

The language of the common core differs from traditional standards. Familiarity with section names and their functions will help you to make the best use of the Common Core State Standards.

Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



Source: Common Core State Standards for Mathematics (www.corestandards.org)

As districts develop units of study for a grade level, careful consideration should be given to the order of, and the connections among, topics and standards. For example, as kindergarten students are learning to rote count within a range of numbers (20 to 40), K.CC.A.1, they will also be counting objects to develop an understanding of number within a different range (1 to 10), K.CC.B.4a, at the same time.

The Common Core State Standards for Mathematical Practice describe eight habits of mind teachers must incorporate into classroom instruction to develop depth of understanding of critical mathematical concepts in their students. The mathematical practices are not intended to be taught in isolation but should be integrated into daily lessons. Some lessons may focus on developing one or two of these standards, while others may incorporate seven or all eight standards. Note that you do not “teach” these standards. Rather, they are the type of mathematical thinking and doing that we want students to practice as they are developing mathematical understanding.

Throughout each domain, we have included examples of mathematical practice that can be used in each cluster. These are not meant to limit lessons to using only those practices. They are examples of key practices that should be included in lessons around that particular cluster. It is likely that you will use all of the practices throughout the cluster and domain.

These eight practices, briefly explained here, are essential for student success. If students are actively engaged in using the eight practices, they are learning rigorous, meaningful mathematics.

SFMP 1. Make sense of problems and persevere in solving them.

Students work to understand the information given in a problem and the question that is asked. They plan a solution path by choosing a strategy they can use to find a solution and check to make sure their answer makes sense. At the K–2 level, students will likely use manipulatives and pictures to conceptualize and solve a problem. Students should be encouraged to find multiple approaches to solve problems.

SFMP 2. Reason abstractly and quantitatively.

Students make sense of quantities and their relationships in problem situations. They develop operational sense by associating contexts to numbers (thinking about $4 + 7$ in a context of putting together 4 pencils and 7 pencils) and associating mathematical meaning to given contexts (eating 4 of 11 strawberries can be expressed as $11 - 4$). Students at the primary level often need to manipulate concrete objects to make meaning of numbers and find solutions.

SFMP 3. Construct viable arguments and critique the reasoning of others.

K–2 students should have opportunities to explain their thinking, justify, and communicate their conclusions both orally and in writing. Listening to others and finding how their strategies are similar may take prompting questions from the teacher such as “Why do you think that works?” or “How is your method the same as . . .?” Mathematical discussions should be a common expectation in mathematics lessons. It will take time for students to become comfortable explaining their thinking, and this will develop over time. Explaining one’s thinking helps to develop deeper conceptual understanding.

SFMP 4. Model with mathematics.

Students use various representations, models, and symbols to connect conceptual understanding to skills and applications. Students in the early grades represent mathematics situations using objects, pictures, numbers, and words. Problem-solving strategies, such as draw a picture, make a list, or write an equation, have explicit connections to representations and models and can be developed at the same time.

SFMP 5. Use appropriate tools strategically.

Students consider the available tools when solving a mathematics problem. In Grades K–2, students will begin with manipulatives before they draw pictures and move to pencil and paper. Students in K–2 also begin to think about appropriate measurement units, moving from informal to standard units to measure various items.

SFMP 6. Attend to precision.

Students communicate precisely with others. K–2 students explain their thinking using the appropriate mathematical vocabulary. At this level, students are beginning to use mathematical symbols that should explicitly connect to vocabulary development.

SFMP 7. Look for and make use of structure.

Students look closely to find patterns and structure in their mathematics work. For example, primary students begin to develop methods to compose and decompose numbers as well as recognize properties of operations. Place value and the commutative property are especially important concepts to develop at this level, as they are foundational to developing operation sense and skill.

SFMP 8. Look for and express regularity in repeated reasoning.

Students notice if calculations are repeated and begin to make generalizations. By recognizing what happens when adding and subtracting tens, students develop a deeper understanding of place value in the early grades and later extend this understanding to what happens when adding and subtracting hundreds. Although this standard mentions shortcuts, it should be noted that shortcuts are only appropriate when students discover them through making generalizations and understand why they work.

Effective Teaching Practices

Quality mathematics teaching is a critical key for student success. In *Principles to Actions* (2014), the National Council of Teachers of Mathematics outlines eight effective teaching practices every teacher should incorporate into their instruction to guarantee student achievement. These eight research-informed practices, briefly explained here, provide a foundation for effective common core mathematics teaching and student learning.

1. Establish mathematics goals to focus learning.

Establishing learning goals sets the stage and helps to guide instructional decisions. Teachers must keep in mind what is to be learned, why the goal is important, where students need to go (the trajectory), as well as how learning can be extended. Students must clearly understand the purpose of each lesson beyond simply repeating the standard.

2. Implement tasks that promote reasoning and problem solving.

Implementing tasks that promote reasoning and problem solving provides opportunities for students to engage in exploration and encourages students to use procedures in ways that are connected to conceptual understanding. The tasks teachers choose should be built on current student understandings and have various entry points with multiple ways for the problems to be solved.

3. Use and connect mathematical representations.

Using and connecting representations leads students to deeper understanding. Different representations, including concrete models, pictures, words, and numbers, should be introduced, discussed, and connected to support students in explaining their thinking and reasoning.

4. Facilitate meaningful mathematical discourse.

Facilitating meaningful student mathematical conversations provides students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. Talking and sharing aloud can advance the mathematical thinking of the whole class.

5. Pose purposeful questions.

Posing purposeful questions reveals students' current understanding of a concept and encourages students to explain, elaborate, and clarify thinking. Asking good questions makes the learning of mathematics more visible and accessible for student examination.

6. Build procedural fluency from conceptual understanding.

Building procedural fluency from conceptual understanding based on experiences with concrete representations allows students to flexibly choose from a variety of methods to solve problems.

7. Support productive struggle in learning mathematics.

Supporting productive struggle in learning mathematics is significant and essential to learning mathematics with understanding. Productive struggle allows students to grapple with ideas and relationships. Giving young students ample time to work with and make sense out of new ideas is critical to their learning with understanding.

8. Elicit and use evidence of student thinking.

Eliciting and using evidence of student thinking helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. Formative assessment through student written and oral ideas are excellent artifacts to assess student thinking and understanding.

More information on these practices can be found in Table 5 in the Resources section.

How to Use This Book

The purpose of this book is to help teachers more deeply understand the mathematical meaning of each cluster and standard within the five domains of Grades K–2. We want this book to be your toolkit for teaching the mathematics standards, and we have left ample space for you to take notes, add ideas, and note other resources you have found to be helpful.

You will find each part is made up of one domain and begins with an overview of how the domain progresses across kindergarten, first grade, and second grade. A list of helpful materials, reproducibles, and key vocabulary from the domain is included in the overview as well.

We track each domain across kindergarten, first grade, and second grade, introducing each cluster and the standards within that cluster. A description of the cluster and how the standards for mathematical practice can be incorporated into your teaching of the cluster concepts follows. Since the standards are intentionally designed to connect within and across domains and grade levels, a list of related standards is included in the cluster overview. We suggest that as you prepare work on a cluster, you look at these standards to have a better idea of the mathematics students learned in previous grades and where they are going in future grades. A list of all of the standards is found inside the front and back covers.

Each standard within a cluster is explained with an example of what the teacher does to work with that standard in the classroom followed by what the students do. It is important to note that many standards will take several days, and it is likely that you will be connecting across standards and domains as you teach for understanding.

Addressing student misconceptions and common errors in developing student understanding of a concept concludes the contents for each standard.

A sample planning page for each grade-level domain provides an example for planning instruction for one standard. Additionally, a template for planning instruction is included at the conclusion of each grade-level domain.

In the resources section, you will find Tables 1–5, which are fundamental to the Operations and Algebraic Thinking and Number and Operations in Base Ten domains. You will also find reproducibles for key materials. These are designed to be samples, and we encourage you to use them or redesign them to best meet the needs of your students. A list of our favorite resource books and high-quality online resources that are particularly useful to developing mathematical ideas in K–2 are also included in the resources section.

We believe that this can become your common core bible! Read it and mark it with questions, comments, and ideas. We hope that it will help you to use these standards and good teaching practice to lay the essential foundation that will ensure your students success not only in your grade but also in all of their future study of mathematics.

Reflection Questions

1. How are the three instructional shifts called for by the common core similar to your current instructional practice? What is conceptual understanding? How is it different from procedural skills? What do you need to consider to teach for conceptual understanding? How can you connect conceptual understanding to help students develop procedural skills? How does the information in Table 1 on problem situations support the development of conceptual understanding?
2. The Standards for Mathematical Practice describe the habits of mind that students need for thinking about and doing mathematics. While not every standard will be in every lesson, select one standard at your grade level and consider some ways you can incorporate these practices in a lesson for that standard. How will these practices provide you with information about student understanding? How will this help you to better assess students? How will this information help you in planning lessons?
3. The Effective Teaching Practices from *Principles to Actions* (2014; NCTM) describe specific actions that teachers must consider in planning and implementing lessons and assessing student performance. How are these practices connected? Work with colleagues to plan a lesson that employs all of these practices. What needs to be considered as you consider goals for the lesson? How can you modify a traditional task so that it promotes reasoning and problem solving? What representations will help students more deeply understand the concept? What questions will you ask students? How will you connect the conceptual understanding to build procedural fluency? What questions will support students who are working to make sense of a new idea? What kind of information will you look for to help inform your instruction? (For more information on the Effective Teaching Practices, go to www.nctm.org.)