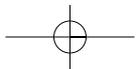
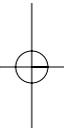
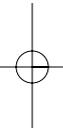


PART I

**How to help pupils stop
counting in ones**



CHAPTER ONE

More than 50 ideas to help pupils stop counting in ones

Overview

Many pupils who struggle with arithmetic have a tendency to count in ones. What is a normal stage of development for most children becomes a crutch for pupils with poor number sense. Pupils who continue to rely on this unsophisticated and laborious strategy well beyond the stage at which counting is appropriate or efficient have fallen into the 'counting trap'.

The counting trap is the situation in which pupils know very few arithmetic facts for certain, and therefore have to calculate every new fact from scratch. They calculate by counting in ones, an arduous and long-winded process that puts so much strain on their already weak memories that the newly found answer becomes dissociated from the question and therefore cannot be added to the store of known facts. Which, in turn, means that very few number facts can be instantly recalled or relied upon, so that every new fact must be calculated afresh.

In order to help such pupils make progress, it is essential to teach them to replace their ones-based approach with chunking techniques. The aim is to minimise the number of calculation steps in order to increase a pupil's chances of achieving a correct solution in a reasonable amount of time and without putting any undue strain on working memory. Eddie Gray, writing in *Teaching & Learning Early Number*,¹ explains how a reliance on counting inhibits flexibility and puts forward the idea that in order for newly calculated facts to be laid down in long-term memory the counting process must be compressed.

Pupils who habitually count on their fingers need to be given lots of opportunities to work through targeted activities that help them become thoroughly familiar with the number bonds of the first ten whole numbers, and then, by extension, of all the whole numbers up to twenty. Pupils must also engage in activities that encourage them to partition numbers into suitable components and to manipulate the components, rather than collections of single units, as they work towards a solution. The objective is for pupils to replace their immature habits with more efficient methods. However, pupils will, understandably, be reluctant to relinquish their well-established counting habits before they feel absolutely secure about any new approach. It is only after plenty of practice, therefore, that pupils will begin to accept that working with components is better than working with ones.

OVERCOMING DIFFICULTIES WITH NUMBER

Games and puzzles are by far the most enjoyable way of getting the necessary practice in component work. This chapter contains more than fifty suggestions for suitable games and activities designed to appeal to adolescent learners. Pupils can be introduced to the games alongside their work on the bridging techniques described in Part II, and should, of course, accept that counting in ones is forbidden during play.

The ideas in this chapter encompass a variety of teaching games and activities targeted at various stages of learning. Included are games that exploit representations of discrete materials arranged into patterns, such as dominoes and dice, games that incorporate continuous concrete materials, such as Cuisenaire rods, and purely abstract activities and puzzles that require only cards or paper and pencil. Pupils should be encouraged to play games from each category. The abundance and variety of the ideas enable new games to be introduced frequently, together with new variations of familiar games, so as to provide regular revision without undue repetition.

The ideas can be used with individuals or with groups of pupils who are being withdrawn from class in order to help them understand the essential arithmetic groundwork without which they will not be able to access the rest of the maths curriculum.

The ideas are also ideal for extra-curricular maths clubs.

Summary of the component games in this chapter

Component game	Number of players	Equipment required
Whose Number Wins?	2	4 sets of domino cards* (20 cards each)
Snap	2	4 sets of domino cards* (20 cards each)
Pelmanism	2	2 sets of domino cards* (20 cards total)
All in a Row	1, 2 or 3	1, 2 or 3 sets of domino cards* (10 cards each)
3-in-a-row Key Components	2	Paper and pencil, one 6-sided die
Basic Domino Game	2 or more	Set of 28 dominoes
Make a 1–10 Sequence	1, 2 or 3	Set of 28 dominoes
Threes and Fives	2	Set of 28 dominoes
Round the Spot	Any number	Three 6-sided dice
Centennial	Up to 5	Three 6-sided dice
Triples Addition	2 or 3	Three 6-sided dice, or 10- or 20-sided dice
Odds or Evens	2 or 3	Three 6-sided dice, or 10- or 20-sided dice
Stuck in the Mud	2 or 3	Five 6-sided dice
Three of a Kind	2 or 3	Five 6-sided dice
Who Has the Last Word?	2	Cuisenaire rods, one 10- or 20-sided die
The 3-Component Challenge	Any number	25 specified Cuisenaire rods each
Cherry Picking	2	Cuisenaire rods, one 10-sided die
Rods in Blocks	2 or 3	Cuisenaire rods, specially labelled die, paper
Decent	2 or 3	Cuisenaire rods, 6-sided die, paper tray
Tens Away	1	pack of playing cards or digit cards*
Eleven Up	1	pack of playing cards
Thirteens & Fifteens	1	pack of playing cards

MORE THAN 50 IDEAS TO HELP PUPILS STOP COUNTING IN ONES

Fifteen in a Suit	1	pack of playing cards
Standing Aces	1	pack of playing cards
Pyramid Solitaire	1	pack of playing cards
Prisoners	1	pack of playing cards
Pontoon	3 or more	pack of playing cards
Zero Blackjack	3 or more	pack of playing cards
Shut the Box	2 or 3	playing cards or digit cards,* two 6-sided dice
Banking Tens	2 or 3	One or two packs of digit cards*
Marching On/Marching Back	2	playing board,* digit cards,* spinner
Conjure the Number	2	pack of digit cards*
Subtract from 15	2	pack of digit cards*
Minimise the Difference	2	pack of digit cards,* paper and pencil
Maximise the Difference	2	pack of digit cards,* paper and pencil
Plus or Minus	2	Plus or Minus cards* pre-prepared by pupils
Magic Squares & other Puzzles	Any number	Paper and pencil
Component Su Doku	Any number	Su Doku puzzles at various levels,* pencil

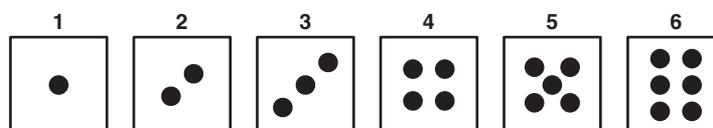
* Can be printed off from the CD.

Dice and domino patterns

Why use them?

Amounts that are otherwise too large to subitise (that is, quantify at a glance) can be read without having to count in ones if the discrete items are arranged in a visually recognisable pattern. Dice and domino patterns are visual patterns that can be easily recognised and are well known by most people.

The dice patterns from 1 to 6 are shown here. It does not matter if the 2 and 3 are sometimes represented vertically or horizontally, rather than diagonally. Amounts as small as these can be readily subitised; numbers above 4 cannot usually be subitised, except when the units are arranged in recognisable patterns.



These patterns can be extended for numbers up to 10 by making doubles patterns for the even numbers and near-doubles patterns for the odd numbers. Doubles patterns are patterns that highlight two identical components, i.e. showing 8 as being built from two patterns of 4, rather than from, say, 6 and 2.

OVERCOMING DIFFICULTIES WITH NUMBER

Some numeracy activities based on these dice patterns can be found in my book *The Dyscalculia Toolkit*.² Many other ideas for young children can be found in *Dyscalculia Guidance* by Brian Butterworth and Dorian Yeo.⁵ A variety of commercially produced games, such as board games in which dice are used, or the dice game Yahtzee, can provide practice in subitising and dot pattern recognition. Several quick and simple dice games suitable for older pupils are described below.

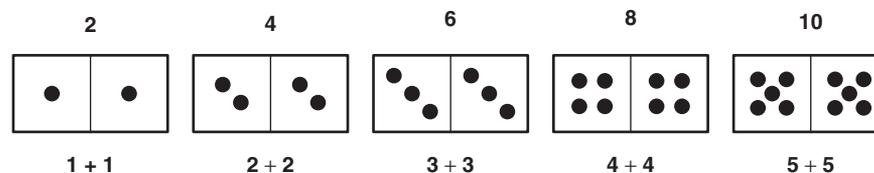
Pupils of secondary school age who perceive dot patterns as childish often prefer to work with dominoes. Dominoes feature in several of the activities, puzzles and games below. As well as clearly showing the doubles and near-doubles components of the numbers up to twelve, dominoes are a fruitful resource because they also show other ways of splitting numbers into paired components.

Activity

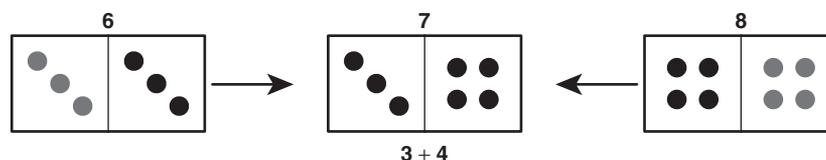
Make domino pattern cards for the numbers 1–10

Pupils should start by building all the even numbers up to 10, by arranging counters or nuggets in doubles patterns on top of a domino background that has been created by splitting a rectangle into two squares or oblongs. Pupils can then make their own set of domino cards by sticking small round labels onto rectangles of card, or by drawing spot patterns. Onto paper or card.

Pupils should be able to read these domino patterns in two ways, both as a total and as a doubles fact. For example, they should recognise the last card shown here as '10' and also as '5 and 5'.

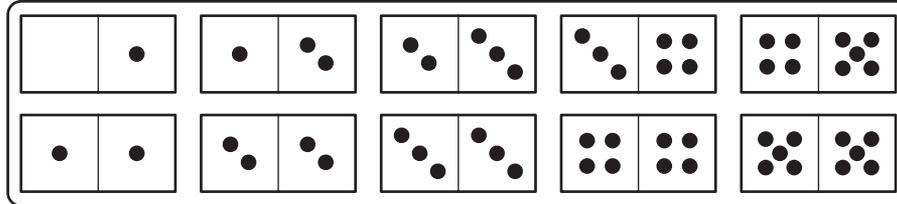


Next, have pupils build all the odd numbers up to 10 out of counters or nuggets, by replicating one of the two patterns from each of the adjacent doubles layout. For example, when making the number 7, position the new domino outline between the 6 and 8 patterns and build the new pattern out of half the 6 and half the 8, i.e. 3 + 4.



MORE THAN 50 IDEAS TO HELP PUPILS STOP COUNTING IN ONES

Pupils should make cards for all five of the even numbers first, and then all five of the odd numbers.



You will find domino pattern cards on the CD  ready to be printed off, but be aware that pupils can learn a great deal from making their own cards.

Play component games with domino cards

Two of the games below can be played with one set of 10 cards for each player. Other games require two sets of cards for each player.

Whose Number Wins?

Two players play with two sets of domino cards each, i.e. 20 cards each.

Rules: Players turn over one domino card at a time, simultaneously, from their own shuffled pack. Each player reads aloud the total number of spots on his/her own card. The player with the higher number wins both cards. If both cards are the same, they remain on the table to be appropriated by the winner of the next round.

Variation: Play so that the lower number wins.

Snap

Two players play with two sets of domino cards each, i.e. 20 cards each.

Rules: Players turn over one domino card at a time, simultaneously, from their own shuffled pack. If both cards show the same total number of spots, the first player who calls out 'snap' wins all the face-up cards.

OVERCOMING DIFFICULTIES WITH NUMBER

Pelmanism

Two players play with two sets of domino cards, i.e. 20 cards in total, shuffled and spread out face down on the table.

Rules: Players take turns to turn over and look at the faces of any two domino cards, which remain face up on the table for the other player to see while the total quantity of spots on each card is read aloud. If both total numbers are the same, the player removes and keeps the cards, and goes on to have another turn. If the card totals do not match, the player turns them both face down again in their original positions and play passes to the next player.

All in a Row

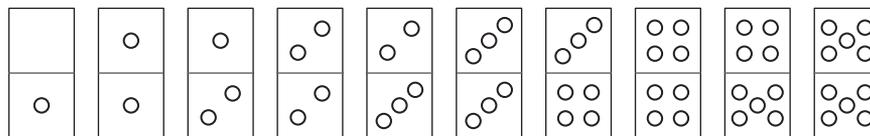
Two or three players each play with a set of 10 domino cards. Alternatively, it can be played as a solitaire game in which players try to beat their previous record.

Rules: Each player shuffles their 10 domino cards and turns over the top five cards, arranging them in order of magnitude. The idea is to see how many cards show adjacent numbers. Score two points for a run of 4 adjacent numbers and ten points for a run of 5 adjacent numbers. The winner is the player with the highest score after 5 rounds.

3-in-a-row Key Components

Two players each sketch a set of domino patterns of the doubles and near-doubles facts, on squared paper, leaving the spots as empty circles, as shown below.

N.B. Do not be tempted to provide a ready-made template for this game, as the preparation makes pupils actively notice the doubles and near-doubles patterns.



Rules: Players take turns to throw a die. A throw of 6 results in the player missing the turn. throw of any other number allows the player to shade in the matching spots in any one place that the pattern appears. For example, if your dice throw is 4, you may choose to colour in the pattern of four on one half of the 8 domino, or on one section of the 7 or 9 dominoes. The winner is the first player to shade in all the spots on three consecutive dominoes, i.e. the first to complete 3 numbers in a row.

Play component games with a set of dominoes

A full set of European dominoes has 28 tiles or stones, from double-zero to double-six, with all the combinations in between.

Whose Number Wins, Snap, Pelmanism, All in a Row

These four games, described above as games to play with domino cards, can also be played with real dominoes. Whereas playing with the cards results in pupils focusing on the doubles and near-doubles component facts, playing with a full set of dominoes provides practice in the other component facts.

Basic domino game

This traditional basic domino game is for two or more players using a full set of 28 dominoes.

Rules: Players take the number of stones that results from the calculation '8 minus the number of players'. After an initial play of a double (6–6 takes precedence, then 5–5, etc.), take turns to place stones in a single straight line, matching the number of spots touching an adjacent stone. You must take a stone from the boneyard if you cannot play and must continue until you can play, provided that at least two stones remain in the boneyard until the end of the game. Play stops when either a player has used all his/her stones or no one can place a domino. The number of dominoes remaining in each player's hand is counted and awarded to the player who finished first. The winner has the highest score after 3 or 5 rounds.

Variations: Another way of scoring is for all the spots on each player's remaining dominoes to count against them, in which case the winner has the lowest score after 3 or 5 rounds. One common variation of the game is for any player placing a double to get an extra turn; another is to dispense with the boneyard and share all 28 stones between 4 players.

Make a 1–10 Sequence

This game for two or three players is played with a full set of 28 dominoes. Alternatively, it can be played as a solitaire game in which players try to beat their previous record.

Rules: Players turn over dominoes one at a time and keep a record of how many they turn over before uncovering enough stones to represent each of the numbers between 1 and 10 inclusive. The winner has the lowest score after 5 rounds.

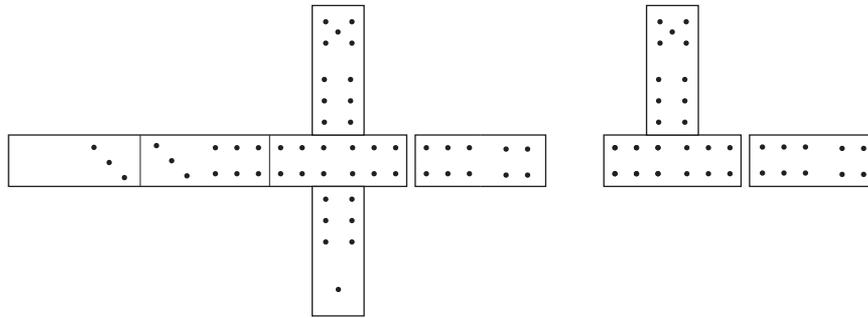
OVERCOMING DIFFICULTIES WITH NUMBER

Threes and Fives

In this traditional domino game, two players use a full set of 28 dominoes.

Rules: Players take 7 stones each. After an initial play of a double (6–6 takes precedence, then 5–5, etc.), take turns to place stones in a cross shape, with stones allowed from the middle of this first stone only, in both directions as well as from the ends. If you cannot play on your turn you must take one or, if necessary, two new stones from the boneyard, which you must play immediately if you can, or add to your stock if you cannot. The aim is to place a stone so that the free ends add up to a multiple of three or five, in order to score the amount of the multiple. Although stones may be placed from the middle of the 6–6 domino, only the patterns at the ends of the dominoes count towards a total.

For example, in the game shown below at the left, the player who places the 6–1 stone scores 4, because the free ends total 12, and the player about to place the 6–4 stone will score 2, because the free ends will total 10. In the game at the right, the total of the free ends will be 15 after placement of the 6–4 stone resulting in a score of 8 (since 15 is a multiple of both 5 and 3). As soon as one player has used all his/her stones the game is over, and the spots on the other player's remaining stones are added to count as penalty points.



The five stones at the left have been played, scoring the last player 4 (i.e. $12 \div 3$). The next player places the 6–4 stone at the right for a score of 2 (i.e. $10 \div 5$).

Placing the 6–4 stone in this game produces a total of 15, scoring 8 (i.e. $15 \div 3 + 15 \div 5$).

Play component games with dice

Five of the following six dice games are addition games. Players may not find their scores by counting up in ones, on their fingers or otherwise, since this is exactly what we are trying to eradicate by encouraging pupils to play these games.

Round the Spot

Any number of players can play this traditional dice game with three 6-sided dice.

Rules: Players take turns to throw all three dice at once. The name, 'Round the Spot' refers to the dice patterns that are clustered around a central spot, i.e. the odd numbers. A number 5 scores four points (four 'petals' around the central spot), a 3 scores two points; however, note that a 1 scores one point. After every turn, each player adds and records his/her own score.

MORE THAN 50 IDEAS TO HELP PUPILS STOP COUNTING IN ONES

Even numbers carry no score, but if a throw shows even numbers on all three dice, the player scores double what he/she scored on the previous turn. The game ends when all the players have had four turns each, and the winner is the player with the highest cumulative score.

Centennial

This traditional dice game is for up to five players using three 6-sided dice.

Rules: Each player writes the numbers from 1 to 12 on a piece of paper. Players take turns to throw all three dice at a time. Players cross off the numbers from their list in numerical order. More than one number can be crossed off on a turn, but no number can be crossed off before the previous number has been eliminated. For example, a first throw of 2, 3 and 6 means the player cannot go, while a throw of 2, 3 and 1 allows all three numbers to be eliminated.

Variation: A common version of this game is for players to have to eliminate all the numbers in turn up to 12, and then all the numbers back from 12 to 1. In this variation, a throw of three 6s after the number 11 has been crossed off would allow both 12s to be eliminated at once.

Triples Addition

This simple dice game is for two or three players using three 6-sided dice.

Rules: Each player takes turns to throw the dice as follows: throw all three, leave the die showing the largest amount and throw the remaining pair, leave the die showing the larger amount and throw the other. Find the total of the three dice and record the score. Play now passes to the next player. The winner is the player with the highest total after 5 rounds each.

Variation: For practice in adding larger numbers, use 10-sided or 20-sided dice.

Odds or Evens

This simple addition game is for two or three players using four 6-sided dice.

Rules: Each player takes turns to throw all four dice at once, but before the throw must choose either odds or evens, and announce the choice to the other player(s). On your throw, you may add only the numbers that conform to your stated choice. The winner is the player with the largest total after 5 rounds.

Variation: For practice in adding larger numbers, use four 10-sided or 20-sided dice, or a mixture of the two.

OVERCOMING DIFFICULTIES WITH NUMBER

Stuck in the Mud

This traditional addition game is for two or three players using five 6-sided dice.

Rules: Each player takes turns to throw the dice as follows: take any dice showing the numbers 1 or 2 out of play (they are 'stuck in the mud'), add the numbers on any remaining dice, then throw these remaining dice again and follow the same procedure until all five dice are 'stuck in the mud'. Keep a running total mentally and record the score as a single total at the end of each round. Play then passes to the next player. The winner is the player with the highest total after 5 rounds.

Three of a Kind

This addition/multiplication game is for two or three players using five 6-sided dice.

Rules: Each player takes turns to throw all five dice together. On your throw, if the dice show three identical numbers, you score the total for the three numbers. If four of the dice show the same number, you score not only the total of the four dice, but also an extra bonus of 5 points. If all five dice show the same number you score an extra bonus of 10 points, in addition to the total of the five dice. The winner is the player with the highest total after 10 rounds of the game. Since many rounds will generate zero scores, you may prefer to end the game only when any one player has had 5 scoring rounds.

Cuisenaire Rods**Why use them?**

Although dice patterns and domino spot patterns are good for promoting visual recognition of component facts, the spot patterns are all built from discrete units. Over-reliance on such patterns will inevitably encourage pupils to continue thinking about numbers as composed of quantities of ones, which is precisely what we are trying to discourage. Cuisenaire rods, on the other hand, show each of the numbers 1 to 10 as a single unit. For example, the black rod representing the number seven can not only be measured in ones to demonstrate that it is equivalent to seven ones, but is also a discrete unit in its own right, i.e. one seven.

I find Cuisenaire rods to be an invaluable resource, responsible for many of those 'aha!' moments when suddenly things seem to click into place in the learner's mind. As with all other concrete teaching materials, rods should never be used to find the answer to a calculation mechanically, but, instead, be used as tools for modelling arithmetic thinking. Many ideas for exploring basic number concepts through rods can be found in *The Dyscalculia Toolkit*.² Many more can be found in numerous articles by Professor Sharma.⁴ A short introduction to Cuisenaire rods, first printed in *The Dyscalculia Toolkit*, is reproduced on the CD. 

Play component games with Cuisenaire rods

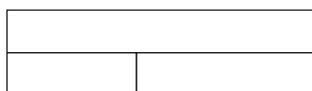
The six games below all require a set of Cuisenaire rods. A single mini-set, sold in a rigid black plastic box with a red lid, contains enough equipment for two players.

Who Has the Last Word?

This game is for two players. The game not only teaches component pairs, but also highlights the fact that there are a limited number of ways in which numbers can be split into components. This is especially striking with smaller numbers. For example each of the numbers 2 and 3 can be split in only one way, i.e. $2 = 1 + 1$ and $3 = 1 + 2$.

Rules: Use a 1–10 die and throw again if the throw is 1. One player starts by throwing the die and taking a single rod to match the throw. Players now take turns to build the same number out of two components. If you are the first player, you must show the double or near-double components of the number by putting two rods end to end below the rod matching the die throw, while explaining the relationship aloud.

For example:



For a dice throw of 7, the first player takes a black rod and places a light green and a purple rod along its length. S/he says, '7 can be built of 3 and 4', or '3 and 4 is 7'.

The next player must make another line of two rods, showing a different pair of components ($4 + 3$ is the same as $3 + 4$, so does not count as a new combination). Play continues until a player cannot make a new row, either because he/she doesn't know the number bonds very well, or because the possibilities have run out. The player who has had the 'last word', gets to keep the last pair of rods until the end of the game. After an agreed number of turns, the winner is the player who has won the largest total amount in rods.

Variation: Play with two ordinary 6-sided dice for numbers up to 12, or with two 10-sided dice for numbers up to 20. Players must still start with the double or near-double fact about the number, and not with the components shown on the two dice.

The 3-Component Challenge

This game, giving practice in combining three components, is played as a race against time between members of a group of any size. Players should each have access to their own allocation of 25 Cuisenaire rods comprising 4 each of the colours white, red, light green, purple and yellow and one each of the remaining five colours to represent the target numbers. No other rods are allowed. Players should also have a screen, e.g. a large book or a book

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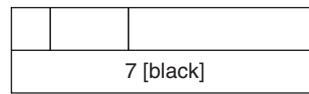
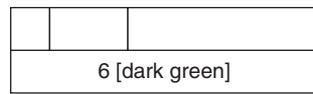
OVERCOMING DIFFICULTIES WITH NUMBER

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file, behind which they can experiment without other players being able to see and copy what they are doing.

The first challenge is to be the first player to succeed in building the five target numbers from 6 to 10 inclusive out of three, and only three, rods.

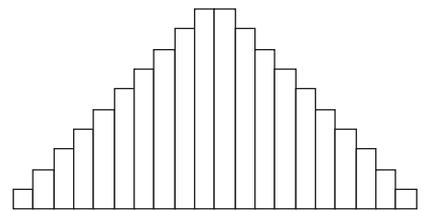
A more difficult challenge introduces the additional constraint that each of the target numbers must be built out of three rods of three different colours. Players soon discover that there is only one way to build the numbers 6 and 7 out of three different components, and will need to experiment with the numbers 8, 9 and 10 to find the unique solution to this second challenge.



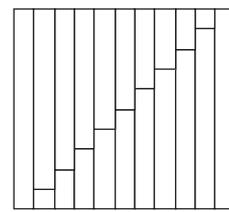
There is only one way to build the numbers 6 and 7 from three different components.

Cherry Picking

This game, for two players, gives practice in building single-digit numbers out of smaller components. As being the first to play gives an advantage in this game, players should take turns to start. Before play begins, the players arrange two of each colour of Cuisenaire rods, in order of size, flat in the lid of the rod box, with the lid placed between the players. The rods can be conveniently arranged either as a double staircase or as a wall, the same arrangement of twenty rods being shared by both players.



Cuisenaire rods arranged in a double staircase.



Cuisenaire rods in a wall.

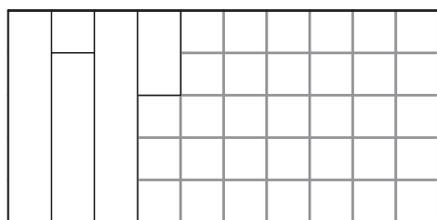
Rules: Players take turns to throw a 1–10 die and take out of the lid as many rods as possible that add up to the exact value of the number thrown. Each rod taken counts towards your score at the end of the game. If you cannot match the dice throw on your turn, you have to miss that turn. The game ends when both players, between them, have to miss three consecutive turns. Score one point for each rod you have removed, irrespective of its length or value. The winner is the player with the highest score after 4 rounds.

Rods in Blocks

This game for two or three players provides practice in splitting numbers greater than 5 into smaller components. Each player prepares for the game by drawing a rectangle measuring 5 cm by 10 cm on 1 centimetre squared paper. Leave a small margin of between half and one centimetre around the edges of the rectangle so that when it is cut out it is possible to fold up and secure the extra margins at the edges to produce a shallow paper tray of 5 × 10 cm.

A more robust alternative is to cut a bespoke shape from craft foam, or packaging foam, leaving a frame of a few centimetres surrounding a rectangular hole measuring 5 by 10 centimetres.

Rules: Players take turns to throw a 6-sided die labelled with the numbers 6 to 11 inclusive. On your turn, you must take rods to match the throw in the most efficient way possible, i.e. taking as few rods as possible, with the aim of eventually blocking out the whole of your rectangle with rods. Place your rods along the side measuring 5 squares and do not start a new column until the previous column is full. For example, an initial throw of 6 must be split into 5 + 1 (a yellow and a white rod) and a second throw of 11 must be split into 4 + 5 + 2 and arranged as shown below. The winner is the first to block out their entire space with rods.



The position in the game Rods in Blocks after two dice throws: first 6 then 11.



A third throw of 7 would need to be split into 3 and 4.

Variation: Use a die labelled 11–16 on a square measuring 10 cm by 10 cm.

Descent

This subtraction game for two or three players provides practice in decomposing tens into units, as well as in partitioning single-digit numbers into component parts. Each player starts by making a shallow paper tray of 5 cm × 10 cm, as described above in the instructions for the Rods in Blocks game. Alternatively, players can use the same foam frame described above in the instructions for the Rods in Blocks game.

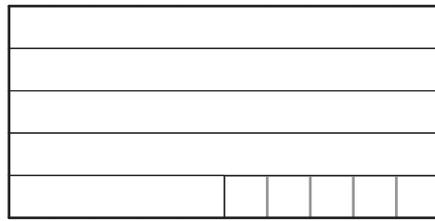
Rules: Players start with five orange rods filling their paper trays or frames. Players take turns to throw a 6-sided die and to remove from their rods the amount that matches the throw. It is important for pupils to work in component chunks in this game, so should not be allowed to exchange rods into single white cubes.

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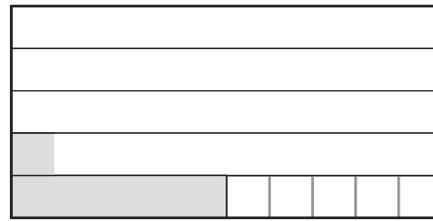
OVERCOMING DIFFICULTIES WITH NUMBER

(Continued)

For example, if your first throw is 3, you must exchange one orange rod for a 3 and a 7, in order to be able to remove 3 (thus practising a complements fact). A subsequent throw of 2 will require the black rod to be exchanged for a 2 and a 5, in order to remove the 2 (thus practising a component fact). A throw of 6 on your next turn will require the 6 to be mentally partitioned into 5 and 1, and the next orange rod to be physically partitioned into 1 and 9, so that the 6 can be discarded in two chunks: 5 and 1 (thus practising decomposition as well as one of the component facts of 6). At the end of the game, the final throw need not be the exact amount (i.e. it can be more). The winner is the first player to empty his/her tray.



The position in the game Descent after 5 has been thrown and removed.



A subsequent throw of 6 would require decomposition before removal.

Variation: Start with rods to the value of 20 or 30 for a shorter game.

Card Games

Play component games with cards

Sixteen card games are presented in this section. The first seven are traditional solitaire games, or adaptations of traditional patience games. They can all be played with a standard deck of playing cards, as can the eighth, ninth and tenth game.

The final six card games in this section have all been invented to target common areas of weakness in arithmetic and are best played with digit cards. Because it is difficult to find commercially produced cards in which the numbers 6 and 9 are easily distinguished, reproducible digit cards are provided on the CD .

Players must not be allowed to count in ones, on their fingers or otherwise, during play. The whole purpose of getting pupils to play the games presented in this chapter is to provide them with practice in chunking, partitioning and manipulating small numbers.

Tens Away

This solitaire game practising the component pairs of 10 is a version of the traditional game sometimes known as Clear the Deck. It is given the name Tens Away when the target number is 10 (see also variations below). The game can be played with a normal pack of playing cards from which the picture cards and the 10s have been removed (an Ace counts as 1), or with a pack of digit cards made of four each of the digits 1–9 inclusive.

Rules: Deal out nine cards face up in a 3 by 3 array. Remove any pair of cards that add up to 10 and fill the empty spaces with new cards from your pack. As you remove the cards name the complement pairs aloud, e.g. say *Seven and three is ten*. The game is won if you can deal out all the cards in the pack.

Variations: To play the traditional Clear the Deck game with a number other than 10 as the target number, remove from the pack all the cards showing the target number and all numbers higher than the target, and lay out an array of one card less than the target. For example, to practise the component of the number 7, play with 4 cards each of the numbers 1–6 inclusive, and set out an array of 6 cards to start the game.

Eleven Up

This is a traditional solitaire game for practising the components of 11. Use a pack of playing cards, counting each Ace as 1.

Rules: Shuffle the cards and set out the first nine cards, face up. Begin to deal out the remaining cards by covering any picture card (Jack, Queen or King) and also any pair of cards that add up to 11. As you play, name the two numbers that combine to make 11. The game is won if you can deal out all the cards in the pack.

Thirteens & Fifteens

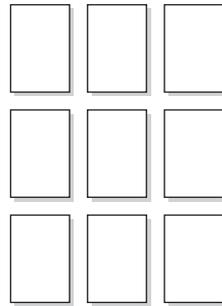
This variant of a traditional solitaire game practises pairs of components of the numbers 13 and 15, with an inbuilt encouragement to construct the larger number. In this game, Kings count as 13, Queens as 12, Jacks as 11 and Aces as either 1 or 14.

Rules: Shuffle a pack of playing cards and lay out the top nine cards, face up. Remove from the array any two cards (except for Kings, see below) that add up to either of the target numbers. Put the card pairs totalling 13 in a winning pile at the left and the card pairs totalling 15 in a winning pile at the right. As soon as cards are removed from the array, fill up the empty spaces with new cards from your pack.

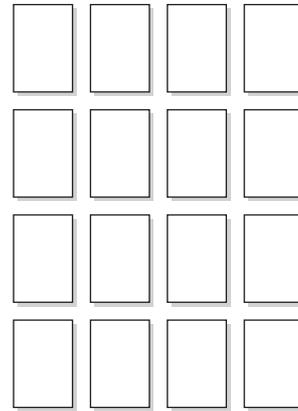
A King may either be removed as a single card and placed in the 13 winning pile, or matched with a 2 and placed in the 15 winning pile. Aces are matched to each other and are put in the 15 winning pile in pairs. The game is won if you can deal out all the cards from your pack and if there are more cards in the right-hand winning pile than the pile at the left, i.e. if you have accumulated more cards totalling 15 than cards totalling 13.

(Continued)

OVERCOMING DIFFICULTIES WITH NUMBER

(Continued)

Card layout for Tens Away, for Eleven Up and for Thirteens & Fifteens.



Card layout for Fifteen in a Suit and for Standing Aces.

Fifteen in a Suit

This traditional solitaire game provides practice in the components of 15. Use a pack of playing cards from which the four 10s have been removed. Count each Ace as 1.

Rules: Shuffle the 48 cards and lay out the first 16 cards, face up. Remove any combination of cards that add up to 15, providing the cards are all of the same suit, and name the number bonds that make 15 as you match and clear the cards. Fill the empty spaces with cards from your pack. As soon as a Jack, Queen and King are all showing, you may remove all three, irrespective of their suits, and fill the empty spaces with new cards. The game is won if you can deal all the cards in your pack and clear the card layout from the table.

Variations: A harder version of this game is to remove the King, Queen and Jack only if they are of the same suit. An easier version of this game is to remove all the picture cards, together with the 10s, at the start.

Standing Aces

This solitaire game provides practice in adding pairs of numbers below 10. Use a pack of playing cards from which the Kings and Queens have been removed. Count each Jack as 11.

Rules: Shuffle the cards and lay out 16 cards, face up, in a 4 by 4 array. Aces cannot be moved or used as part of any addition. Clear the other cards from the array in pairs, but only if both cards are of the same suit and are removed from the same column or the same row of the array. As you clear the cards, you must mentally add the numbers on both cards and

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announce the total of the pair. As soon as spaces appear in the array, fill the gaps with new cards from the pack. The game is won when only the four Aces remain.

Variation: Because it is quite difficult to clear the last few remaining cards, you may like to keep a score of the number of cards that remain on the table, apart from the Aces, and try to beat your record in subsequent games.

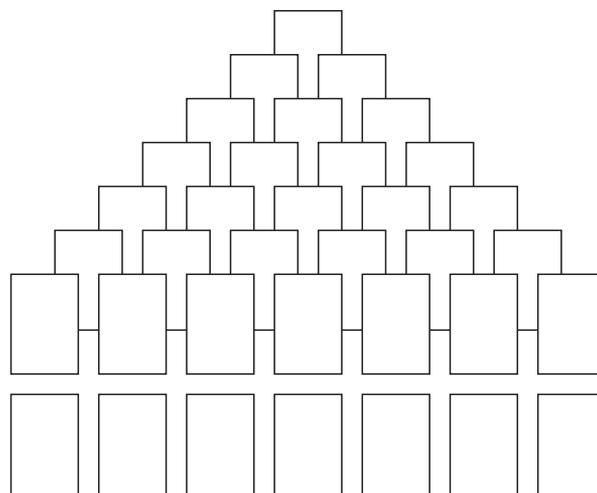
Pyramid Solitaire

This challenging solitaire game provides practice in the components of 13. Use a pack of playing cards and count each Ace as 1, a Jack as 11, a Queen as 12 and a King as 13.

Rules: Shuffle the cards and lay out the first 28 cards, face up, in a pyramid pattern as shown below, with cards overlapping from top to bottom. Set out a further 7 cards below the pyramid in a row so that none of the cards overlap or are overlapped by another. The rest of the pack is kept face down in your hand. The only cards that are in play are those face up and not overlapped by another card.

Look for pairs of card in play that add up to 13. Remove them, and any Kings (worth 13 on their own) that are in play, from the pyramid and set them aside in a discard pile. During play, you must name the number bonds that make 13 as you match and clear the relevant cards. When no more cards can be removed, take the top card from the pile in your hand and try to match it with any face-up card in play to make a total of 13. If you cannot match the card, put it face up onto a reserve pile, from where it can remain in play as long as it is not covered by another card. When you have been through all the cards in your hand, you may turn over the reserve pile and go through these cards again, once only.

The game is won if all the 35 cards in the pyramid layout have been cleared.



Card layout for Pyramid Solitaire.

OVERCOMING DIFFICULTIES WITH NUMBER

Prisoners

This solitaire game gives practice in building the numbers 11, 12 and 13. Use a pack of playing cards, from which the Aces, 2s and 10s have been removed. Count each Jack as 11, Queen as 12 and King as 13.

Rules: Take out the picture cards and place them, face up, at the top of your playing space. These 12 cards represent the 'prisoners' that must be liberated for the game to be won. Shuffle the remaining 28 number cards and deal a row of 6 cards, face up. These 6 cards are the active cards.

The release of one of the 'prisoners' can be 'bought' by putting together any two of the number cards that add up to the worth of the picture card. In other words, a Jack can be freed by two number cards with a total of 11, a Queen can be freed by a pair of cards totalling 12 and a King can be freed by a pair of cards totalling 13. As each prisoner is released, remove the picture card and the two cards paying for its release from the game. Deal new cards from the pack to fill the gaps as they appear in the row of 6 active cards. The game is won when all the prisoners have been released.

Pontoon

This traditional game, also known as Blackjack, Vingt-et-un or Twenty-one, is a game for three or more players including one who is the dealer. Players should take turns to act as the dealer.

Rules: The dealer shuffles a pack of playing cards and deals two cards to each player, face down. Players add their cards, and aim to get the highest score without exceeding 21. Aces can score either 1 or 11. The picture cards each carry a score of 10.

The dealer goes round the players in turn who must each say whether they would like another card, in which case they say 'hit me', or whether they would not like another card, in which case they say 'stick'. A player whose cards total more than 21 says 'bust' and lays the hand face up on the table. A player may ask for a fourth card on the next round, and a fifth or more on subsequent rounds, until all the players have either gone bust or decided to stick. The dealer now turns over his own cards and goes through the same process of 'hit me' or 'stick' for him/herself with the other players looking on.

If the dealer goes bust, all the players who are still in the game win a point. If the dealer sticks at 21 or less, anyone who is closer to 21 wins point. The dealer wins a point if he/she is the closest to 21. If the dealer scores the same as another player, with no one else scoring higher, both win a point.

Zero Blackjack

Zero Blackjack is a version of the traditional game described above as Pontoon. All the same rules apply except that it is played with a pack of cards from which all the picture cards have been removed and that players are limited to a maximum of five cards each. In this version of the game all the black cards are regarded as positive and all the red cards as negative. Players aim for a target score of zero to win the round, or win a point for being closest to zero if no player achieves the target.

Shut the Box

This traditional component game for two or three players is often sold in the form of a smart wooden box game, but can just as easily be played with either playing cards or digit cards and two 1–6 dice.

Rules: Each player sets out and controls a set of nine cards, one for each of the numbers from 1 to 9. On your turn, throw both dice, add the two numbers thrown, and turn over any card or any combination of cards from your own set of numbers to match the total, thus removing those cards from play. In the version of the game I am proposing here, you may not match the dice throw exactly unless you have no alternative. For example, if you throw 2 and 3 on the dice, you may turn over the 5 card, or the 1 and 4 cards, but may only turn over the 2 and 3 cards if neither of the other options is available. Continue to throw and to turn over cards until you no longer have cards in play that can match the dice total. Play then passes to the next player.

The winner is traditionally the player with the fewest cards still in play, i.e. still face up. However, in the version of the game I am proposing here, players score by adding the numbers on their remaining cards. The player with the lowest score is the winner. This scoring system encourages a different strategy because it gives players an incentive to use the larger numbers wherever possible.

Banking Tens

This game for two players requires a pack of 36 digit cards made up of four cards each of the numbers 1–9. The game can alternatively be played by three players with a pack of 45 or 54 cards made up of five or six cards of each digit. The game practises complements to 10 and addition using complement facts.

Rules: Players shuffle the pack and take four cards each to start. On your turn, take a card from the top of the pack to add to your hand, and try to find two cards in your hand that add up to 10. If you are able to do so, display the pair of cards to the other players, saying the two numbers aloud, and then 'bank' the pair of cards, face down on the table. As soon as you bank a pair of cards, you can have another turn and possibly bank another 10 if the new card complements a card already in your hand, and so on. When a player can bank no (more) cards, play passes to the next player.

When all the cards in the pack have been used, players count up the value of their banked cards, which should be kept in their original pairs to aid scoring. Scoring is an important part of the teaching point of this game, so players should be shown how to score by keeping a running total while stating the subtotals aloud at each step. As a first step, players can set the cards out in pairs and count up the running total of the tens: 10 ... 20 ... 30 ... etc. Then, they must point to each card individually and announce the running total: e.g. 3 ... 10 ... 15 ... 20 ... 21 ... 30, etc. The winner is the player with 'the most money in the bank', i.e. the largest value of banked cards.



During the game, cards are kept in their complement pairs, as shown here at the left.
For scoring keep a running total, like this: 3 ... 10 ... 15 ... 20 ... 21 ... 30 ... 38 ... 40.

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OVERCOMING DIFFICULTIES WITH NUMBER

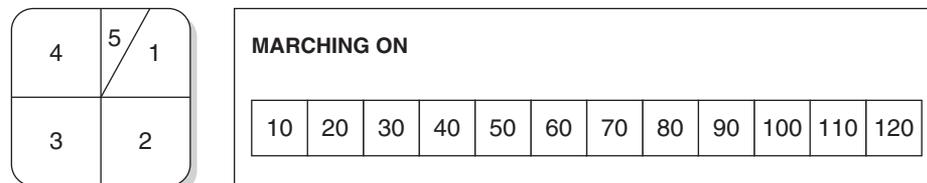
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Variation: Play with digit cards representing the multiples of ten up to 100 with the digits set out in a landscape format, to represent banknotes. A pack is built from four cards each of the round numbers (multiples of ten) between 10 and 90 inclusive. CD . Players put money in the bank as soon as they have a pair of cards adding up to 100. Insist on the same scoring system at the end of the game as described above, so that players practise keeping a running total in which every alternate card brings the running total up to a round hundred.

Marching On

This game is for two players and requires a game board for each and an augmented set of digit cards. Start with a pack of digit cards made of four cards each of the numbers 1–9, then add in all the numbers 1–4 inclusive from a second pack, shuffling all 52 cards together. The game board is made of a track of 12 adjacent rectangles. CD  The game also requires a spinner that is divided into four equal sections labelled 1 to 4 with a small slice taken from the quarter allocated to the number 1 to make space for the number 5.

The game gives practice in building the number 10 from two or more components and also gives practice in counting in tens beyond 100, across the difficult transition points.



The spinner base and one player's board for the game Marching On.
Game boards can be marked with any twelve consecutive multiples of ten.

Preparation: Players each have their own game board, marked with the same set of numbers as each other. An early game might use the board shown here, on which players go 'marching on' from zero to 120; later games might start at 50, or 150, or 970, or any round number (multiples of ten) that provides a few steps of run-up before a difficult transition point.

The CD  also provides a blank format for the game, to allow any suitable range of twelve consecutive multiples of ten to be selected, depending on which crossover points in the counting sequence are difficult for your particular pupils.

Rules: Each player takes it in turn to spin the spinner. On your turn, take as many cards as match the spin. If any combination of cards adds up to 10, display them to your opponent while naming them aloud, then put them face down on top of the first round number (multiples of ten) on your track blocking the round number from view. You must announce this aloud as a movement along the track, e.g. 'I can move from zero to 10', or 'I have put together another 10, so now I can go from 20 to 30'. You can go as many times as your cards allow, but only in steps of ten, before play passes to the next player.

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If the cards run out before either player has reached the end, as can happen if the game is a close one, players can mark their position on the track with a token, and the cards already used can be collected, shuffled and reused. The winner is the first player to reach the end of the track.

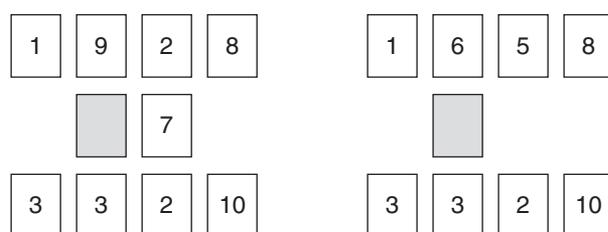
Variation: To play a version of the game called Marching Back, all the digit cards are deemed to represent negative numbers; alternatively make a pack of negative digit cards for the purpose. Players put together two or more cards to make a total of minus 10 for every step that they travel back from right to left, from the end of the track to the beginning.

Conjure the Number

This game for two players is played with a pack of 40 digit cards with four cards each of the numbers 1–10.

Rules: Players shuffle the pack and take four cards each to place face up in front of them. The rest of the pack is placed face down between the players. Players take turns to turn over the top card from the pack and to try to make this target number by either adding or subtracting the numbers on any two of their own cards. If successful, the player wins all three cards and takes two new cards from the pack to fill the gaps. If the player cannot make the target number, the card remains on the table and is covered by the next player's target card. If successful, the next player wins this extra card together with the three cards won on the turn.

For example, in the layout shown below, the player at the top makes the target number by subtracting 2 from 9, wins the three cards (7, 2 and 9) and replaces the 2 and 9 with new cards from the pack. The player at the bottom now turns over a card from the central pile and hopes it will show a new target of 1, 5, 6, 7 or 8.



Two successive snapshots from a game of Conjure the Number.

Variation: Introduce ten extra digit cards into the pack to represent one each of the numbers 11 to 20.

OVERCOMING DIFFICULTIES WITH NUMBER

Subtract from 15 (or from any other teen number)

This subtraction game for two players is played with a pack of 40 digit cards with four cards each of the numbers 1–10.

Rules: Players have five cards each throughout the game, taking new cards from the pack as soon as cards have been played.

Players take turns to lay down any card from his/her hand and challenge the opponent to subtract it from the agreed number and then match the solution with any combination of cards from his/her own hand. If successful, the second player wins all the cards on the table. If unsuccessful, the card is added to the winning pile of the player who laid it down.

For example, if your cards are 9, 9, 4, 3 and 1, you could win cards if your opponent puts down a 1, by playing 9, 4 and 1, and winning these three cards as well as the opponent's 1. Similarly, you could win if your opponent challenges you with a 2 (you would play 9 and 4), a 3 (you would play 9 and 3) a 5 (you would play 9 and 1), (you would play 4, 3, and 1) etc. However, you could win nothing if your opponent puts down a 4 or a 9. Play continues until there are insufficient cards in the pack for each of the players to keep five cards in their hand. The winner is the player who has won the most cards.

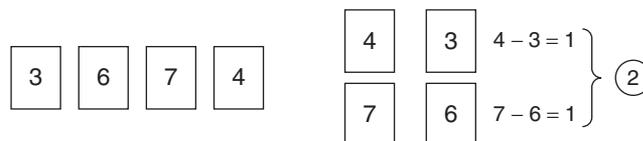
Variations: Choose any teen number other than 15 as an agreed starting point for the game.

Minimise the Difference

This subtraction game for two players is played with a pack of 40 digit cards with four cards each of the numbers 1–10.

Rules: Players take turns to pick up four cards from a shuffled pack. On turning the cards face up, the player must arrange the numbers into pairs in such a way as to minimise the difference between the two pairs of numbers. For example, a player whose cards are 3, 6, 7 and 4 will do better to choose the subtractions $7 - 6$ and $4 - 3$, which will score them a minimal 2, than to choose either of the other alternatives.

The player writes down the two subtraction sums to represent the working, as shown here, and scores the total of the two differences. The winner is the player with the lowest score after 3 rounds.



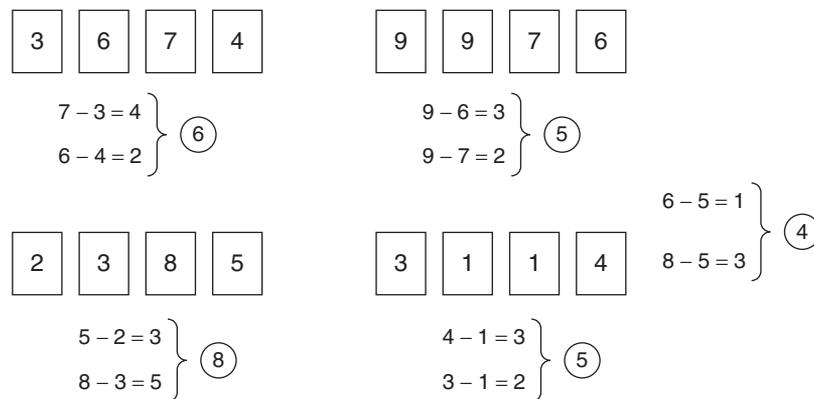
In the game Minimise the Difference, cards are arranged into pairs and recorded as subtractions

Maximise the Difference

This is a subtraction game for two players, which is somewhat harder and more complex than the related game above. It is played with a pack of 40 digit cards with four cards each of the numbers 1–10.

Rules: Players take turns to pick up four cards from the shuffled pack. On turning the cards face up, the player must arrange them into pairs in such a way as to maximise the difference between two pairs of cards. For example, a player whose cards are 3, 6, 7 and 4 will do better to choose the subtractions $7 - 4$ and $6 - 3$ or $7 - 3$ and $6 - 4$, either of which would score 6, than the alternative combination of $7 - 6$ and $4 - 3$, which scores only 2. The player writes down the working as two subtraction sums, and scores the total of the two differences on that round.

After each player has played four rounds, players take their four scores and arrange these four numbers so as to maximise the difference between pairs of the numbers, in just the same way as they did when dealing with the cards. The winner is the player with the highest final score.



In Maximise the Difference, the scores from 4 rounds are arranged and recorded as yet more subtractions to maximise the difference between four numbers. This player's final score is 4.

Other Games and Puzzles

Paper and pencil puzzles that encourage component thinking

Paper and pencil games and puzzles treat numbers as abstractions. The suggestions below, therefore, are recommended only for pupils who have already been given the opportunity to learn about numbers in a concrete way through exploring and manipulating physical apparatus.

It is of the utmost importance to ensure that players are not allowed to count up in ones, on their fingers or otherwise, since this is exactly what we are trying to eradicate by encouraging pupils to engage with these puzzles and games.

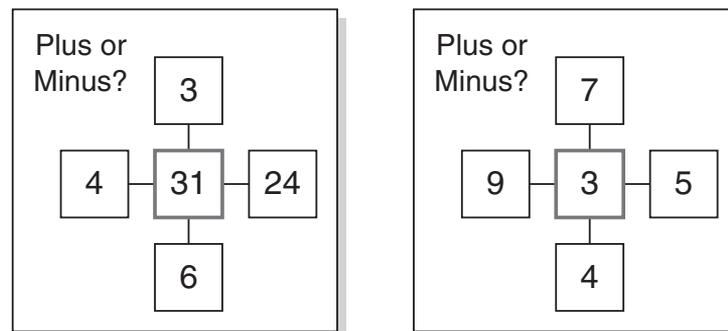
OVERCOMING DIFFICULTIES WITH NUMBER

Make and play a game of Plus or Minus

This game provides mixed practice of mental addition and subtraction. It also reinforces the idea that the steps within a string of additions and subtractions can be carried out in any order, i.e. that $4 + 2 - 3$ is equivalent to $4 - 3 + 2$ or $2 - 3 + 4$.

The game requires pre-prepared cards. The activity of creating the cards will provide pupils with as valuable a learning experience as actually playing the game, if you insist that all calculations are performed mentally. It is a good idea to set a limit on the size of the numbers used when creating the cards, for example by stipulating that none of the numbers can be greater than 30, or declaring that all target numbers must be 20 or less.

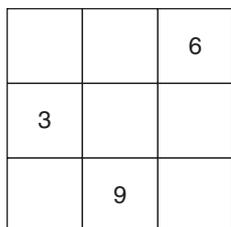
A template for making the cards for Plus or Minus can be found on the CD . Each card shows a number surrounded by four components. All four components must be used once, and once only, to make the target number in the middle. In this activity/game, the operations are limited to plus and minus. For example, on the first card shown below, the number 31 can be created by adding 24, 6 and 4 and subtracting 3. Try the second card for yourself.



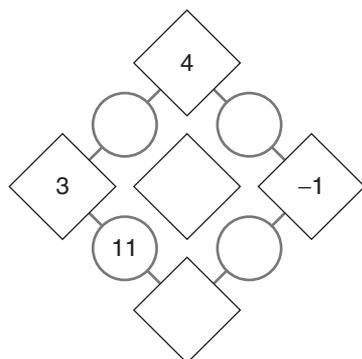
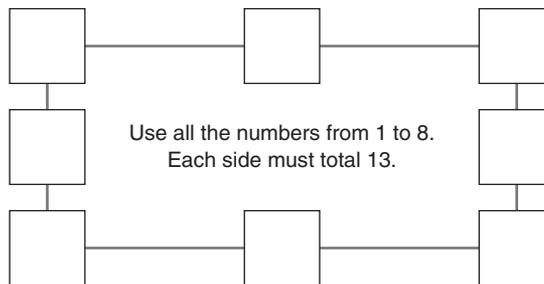
Rules: The game is played with two or three players and a small stack of puzzle cards. Either play so that each pupil has a turn to solve the puzzle within one minute before the same card is offered to the next player, or, if the pupils are evenly matched in ability, play so that each card can be won by the first person to write down a solution.

Magic Squares and Number Puzzles

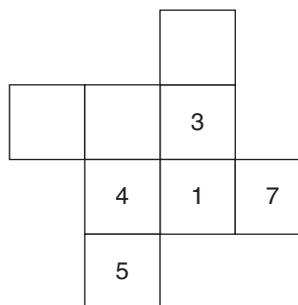
A magic square is one in which all the rows, columns and diagonals add up to the same number, and where each number appears only once. Many similar puzzles are widely available, including on the Internet. Here are four examples.



Use all the numbers from 1 to 9.
Each line must total 15.



Fill in the missing numbers so that the numbers in the circles are the sum of the numbers in the two diamonds on either side.
Add the four corner numbers to complete the central diamond.



Fill in the missing numbers so that each 3-cell column and 3-cell row has the same total.

Component Su Doku

In the Su Doku puzzles shown below the numbers from 1 to 5 can appear only once in each row and in each column. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.

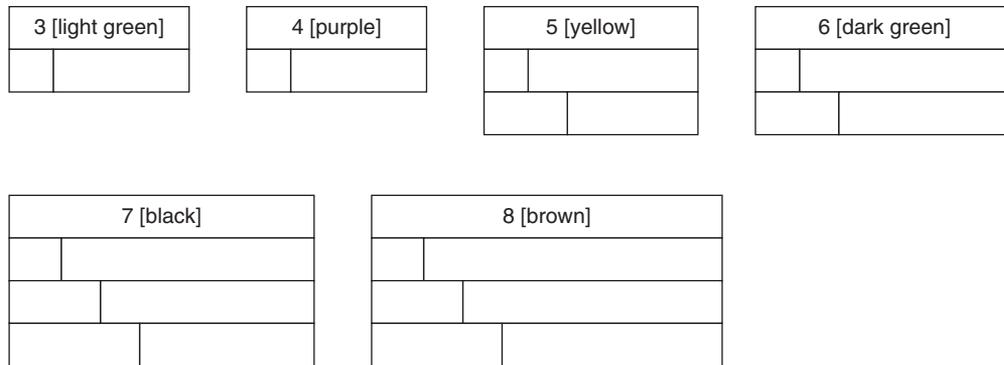
The two component Su Doku puzzles shown here, together with another 22 Su Doku component puzzles at different levels of difficulty, can be printed off the CD. The puzzles at the moderate level can be solved by a process of elimination together with an understanding of the limited possibilities for splitting very small numbers into components. To solve the intermediate puzzles, pupils must also be able to work out the total values in each row and column and use that fact to make deductions. The most difficult puzzles require several different techniques and the versatility to choose the appropriate strategy at each stage of the solution. All the puzzles provide excellent opportunities to practise building up and breaking down numbers into components.

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OVERCOMING DIFFICULTIES WITH NUMBER

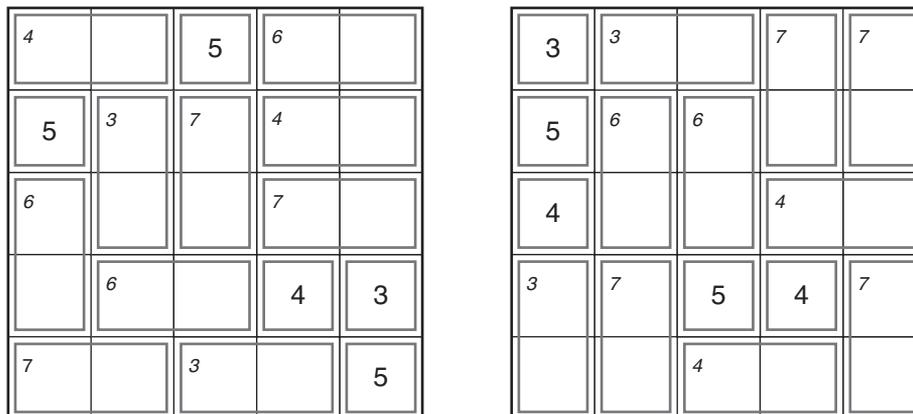
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If you find your pupils trying to solve these puzzles by counting up on their fingers, give them more practice in some of the earlier component games using Cuisenaire rods, such as Who Has the Last Word?, or Descent, or Rods in Blocks. Alternatively, provide the pupils with a set of Cuisenaire rods and insist that they build all the numbers below 8 out of two different components, before tackling the Su Doku puzzles. The rods should stay within view so that pupils can use them for reference. Pupils will be able to see from the rods that there is only one acceptable way of building the numbers 3 and 4 (since the doubles combination of 2 + 2 would break the Su Doku rule of different numbers within each row and column); that the only possible combinations for building 5 are 1 + 4 or 2 + 3; that the only two acceptable ways of building 6 is by pairing 1 + 5 or 2 + 4; that there are only 3 possible ways of building 7 from two components, etc. Discovering how very few combinations are possible often reassures pupils who might otherwise believe that they are expected to memorise an unfeasible number of component facts.



Building small numbers out of two different rods highlights how few possibilities there are, thereby reassuring pupils that there are not too many component facts to learn.

These Su Doku puzzles should always be solved by logic and reasoning, not by guesswork or trial and error.



In these Component Su Doku puzzles the numbers from 1 to 5 appear once in each row and in each column. The thicker lines enclose two different components that add up to the number in the top left corner of the enclosure.

MORE THAN 50 IDEAS TO HELP PUPILS STOP COUNTING IN ONES

Pupils who enjoy the Su Doku puzzles shown here and on the CD  might also like to try the mild versions of the 'Killer Su Doku' puzzles that are published in some daily newspapers.

A variant of the Component Su Doku puzzle is the Difference Su Doku puzzle, in which clues are given about the difference between two components, i.e. the subtraction fact. Two examples are shown below.

Difference of 2	Difference of 4	3
Difference of 1	5	Difference of 3
5	Difference of 2	Difference of 4
	2	Difference of 1
Difference of 4	3	2

5	Difference of 3	3	Difference of 2
Difference of 2	Difference of 3		
Difference of 2	2	Difference of 2	1
	Difference of 1	Difference of 3	Difference of 2
Difference of 4	2		

In these Difference Su Doku puzzles the numbers from 1 to 5 appear once in each row and in each column. The difference between the two components within a single enclosure is noted at the top of the enclosure.

References

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- 4 M. Sharma (various 1980–93) *Math Notebook*, Center for Teaching & Learning of Mathematics, Framingham, MA, USA. (Professor Sharma's publications and videos are available in the UK from Berkshire Mathematics: www.berkshireremathematics.com)

